Lecture 4
2018/2019

Microwave Devices and Circuits for Radiocommunications

2018/2019

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course + laboratory)**
- Lectures- **associate professor Radu Damian**
  - Friday 09-11, II.13
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - 3p=+0.5p
  - **all materials/equipments authorized**
- Laboratory – **associate professor Radu Damian**
  - Wednesday 12-14, II.12 odd weeks
  - L – 25% final grade
  - P – 25% final grade

Materials

- http://rf-opto.etti.tuiasi.ro

Laboratorul de Microunde si Opt ×

Not secure | rf-opto.etti.tuiasi.ro/microwave_cd.php?chg_lang=0

Main **Courses** Master Staff Research Students Admin

Microwave CD Optical Communications Optoelectronics Internet Antennas Practica Networks Educational software

Microwave Devices and Circuits for Radiocommunications (English)

Course: MDCR (2017-2018)

Course Coordinator: Assoc.P. Dr. Radu-Florin Damian
Code: EDOS412T
Discipline Type: DOS; Alternative, Specialty
Credits: 4
Enrollment Year: 4, **Sem.** 7

Activities

Course: Instructor: Assoc.P. Dr. Radu-Florin Damian, 2 Hours/Week, Specialization Section, Timetable:
Laboratory: Instructor: Assoc.P. Dr. Radu-Florin Damian, 1 Hours/Week, Group, Timetable:

Evaluation

Type: **Examen**

A: 50%, (Test/Colloquium)
B: 25%, (Seminary/Laboratory/Project Activity)
D: 25%, (Homework/Specialty papers)

Grades

Aggregate Results

Attendance

Course
Laboratory

Lists

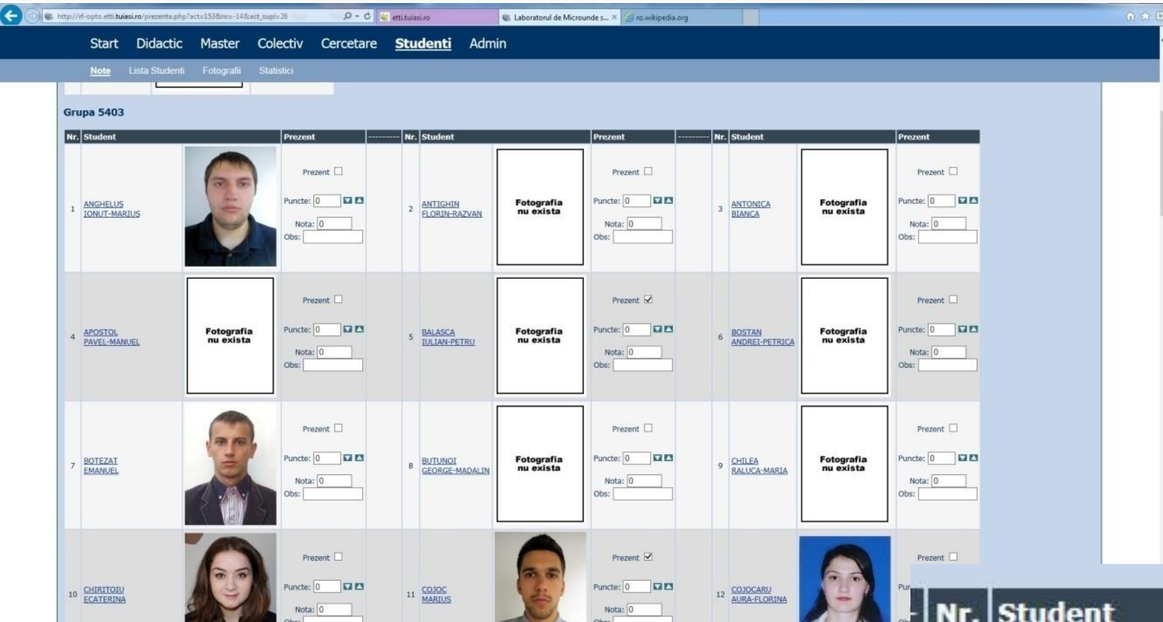
Bonus-uri acumulate (final)
Studenti care nu pot intra in examen

Materials

Course Slides

MDCR Lecture 1 (pdf, 5.43 MB, en,)
MDCR Lecture 2 (pdf, 3.67 MB, en,)
MDCR Lecture 3 (pdf, 4.76 MB, en,)
MDCR Lecture 4 (pdf, 5.58 MB, en,)

Photos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ANTIGHIN FLORIN-RAZVAN
ANTONICA BIANCA
APOSTOL PAVEL-MANUEL
BALASCA IULIAN-PETRU
BOSTAN ANDREI-PETRICA
BOTEZAT EMANUEL
BUTUNOI GEORGE-MADALIN
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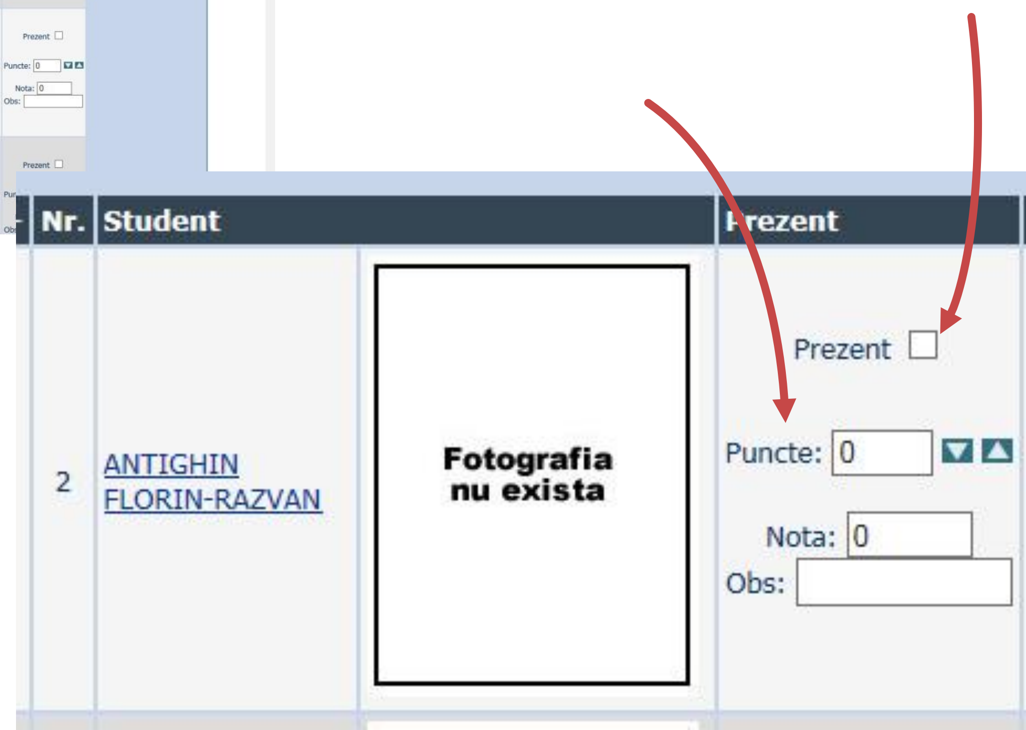
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Access

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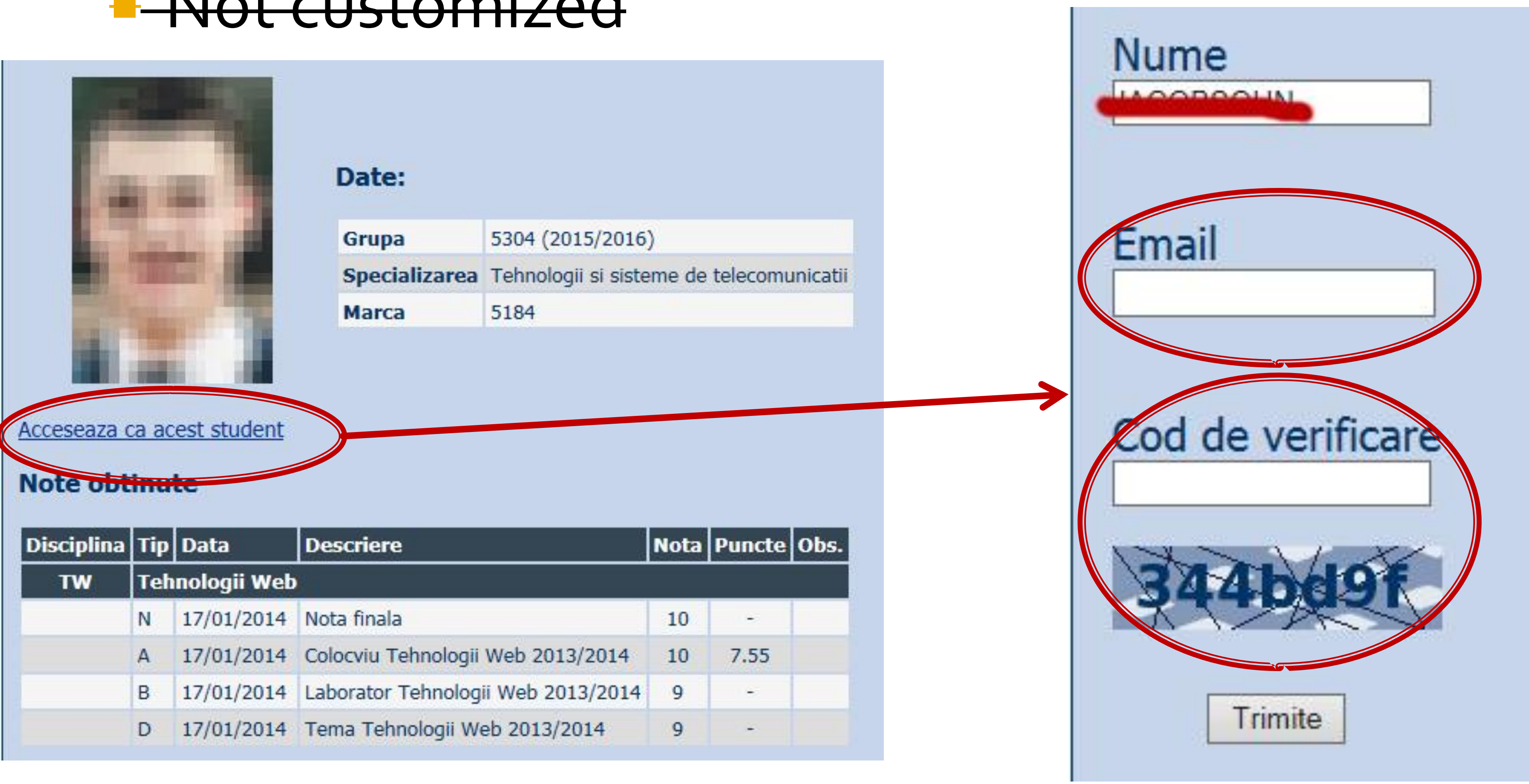


Date:

Grupa	5304 (2015/2016)
Specializarea	Tehnologii si sisteme de telecomunicatii
Marca	5184

Acceseaza ca acest student

Note obtinute

Disciplina	Tip	Data	Descriere	Nota	Puncte	Obs.
TW	Tehnologii Web					
	N	17/01/2014	Nota finala	10	-	
	A	17/01/2014	Colocviu Tehnologii Web 2013/2014	10	7.55	
	B	17/01/2014	Laborator Tehnologii Web 2013/2014	9	-	
	D	17/01/2014	Tema Tehnologii Web 2013/2014	9	-	

Nume

[redacted]

Email

Cod de verificare

344bd9f

Trimite

Examen: Logarithmic scales

dB = 10 • $\log_{10}(P_2 / P_1)$

0 dB	= 1
+ 0.1 dB	= 1.023 (+2.3%)
+ 3 dB	= 2
+ 5 dB	= 3
+ 10 dB	= 10
-3 dB	= 0.5
-10 dB	= 0.1
-20 dB	= 0.01
-30 dB	= 0.001

dBm = 10 • $\log_{10}(P / 1\ mW)$

0 dBm	= 1 mW
3 dBm	= 2 mW
5 dBm	= 3 mW
10 dBm	= 10 mW
20 dBm	= 100 mW
-3 dBm	= 0.5 mW
-10 dBm	= 100 μW
-30 dBm	= 1 μW
-60 dBm	= 1 nW

[dBm] + [dB] = [dBm]

[dBm/Hz] + [dB] = [dBm/Hz]

[x] + [dB] = [x]

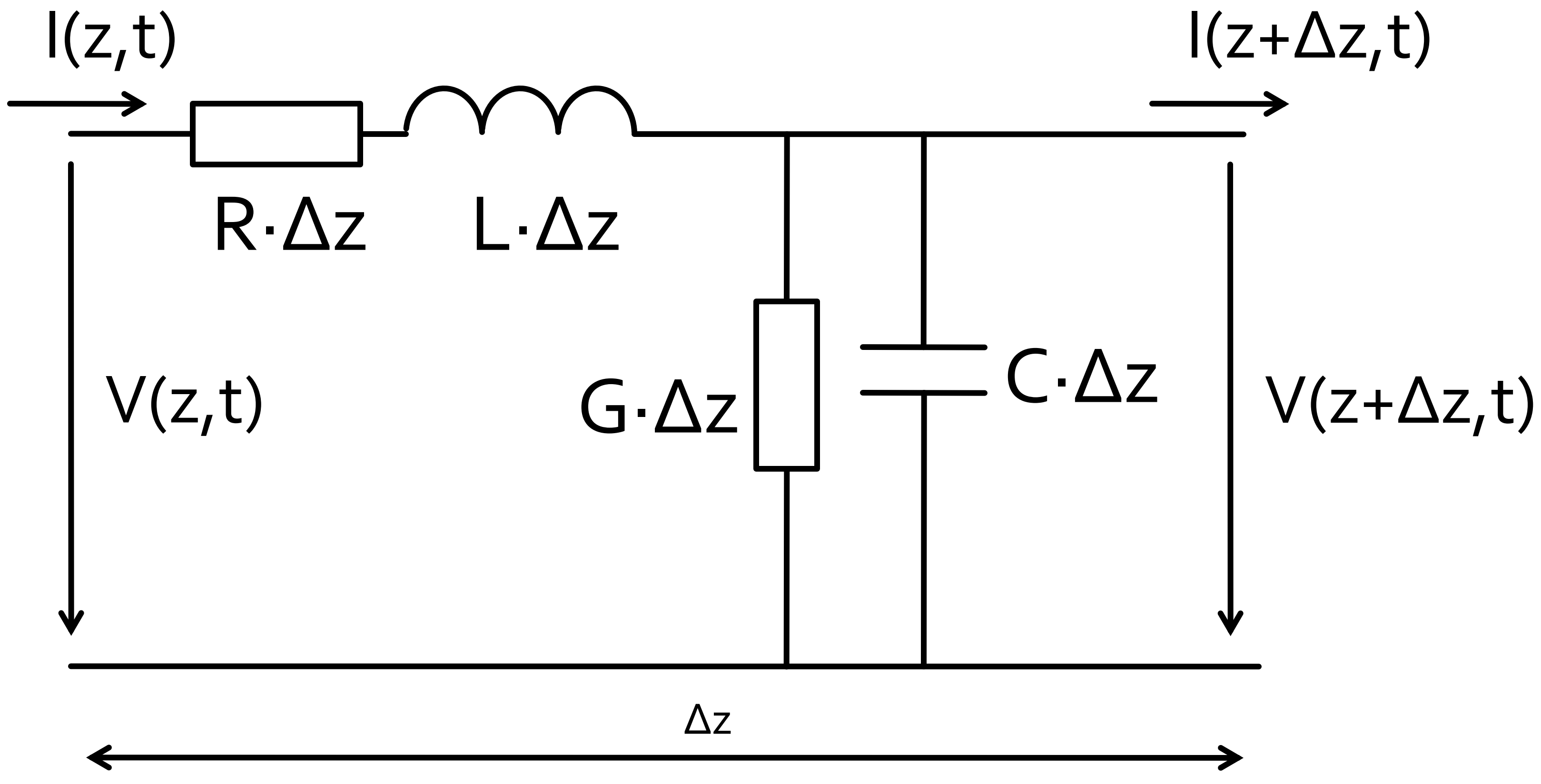
Examen

- Complex numbers arithmetic**!!!!**
- $z = a + j \cdot b$; $\mathbf{j^2 = -1}$

TEM transmission lines

Transmission line equivalent model

- TEM wave propagation, at least two conductors



Solutions

$$\begin{cases} V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z} \\ I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z} \end{cases}$$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z)$$

$$I(z) = \frac{\gamma}{R + j \cdot \omega \cdot L} \left(V_0^+ e^{-\gamma \cdot z} - V_0^- e^{\gamma \cdot z} \right)$$

$$Z_0 \equiv \frac{R + j \cdot \omega \cdot L}{\gamma} = \sqrt{\frac{R + j \cdot \omega \cdot L}{G + j \cdot \omega \cdot C}}$$

- Characteristic impedance of the line

$$\frac{V_0^+}{I_0^+} = Z_0 = -\frac{V_0^-}{I_0^-}$$

$$\lambda = \frac{2\pi}{\beta} \qquad v_f = \frac{\omega}{\beta} = \lambda \cdot f$$

The lossless line

- **Lossless**: R=G=0

$$\gamma = \alpha + j\cdot\beta = \sqrt{(R + j\cdot\omega\cdot L)\cdot(G + j\cdot\omega\cdot C)} = j\cdot\omega\cdot\sqrt{L\cdot C}$$

$$\alpha = 0 \quad ; \quad \beta = \omega\cdot\sqrt{L\cdot C}$$

$$Z_0 = \sqrt{\frac{R + j\cdot\omega\cdot L}{G + j\cdot\omega\cdot C}} = \sqrt{\frac{L}{C}}$$

- Z_0 is **real**

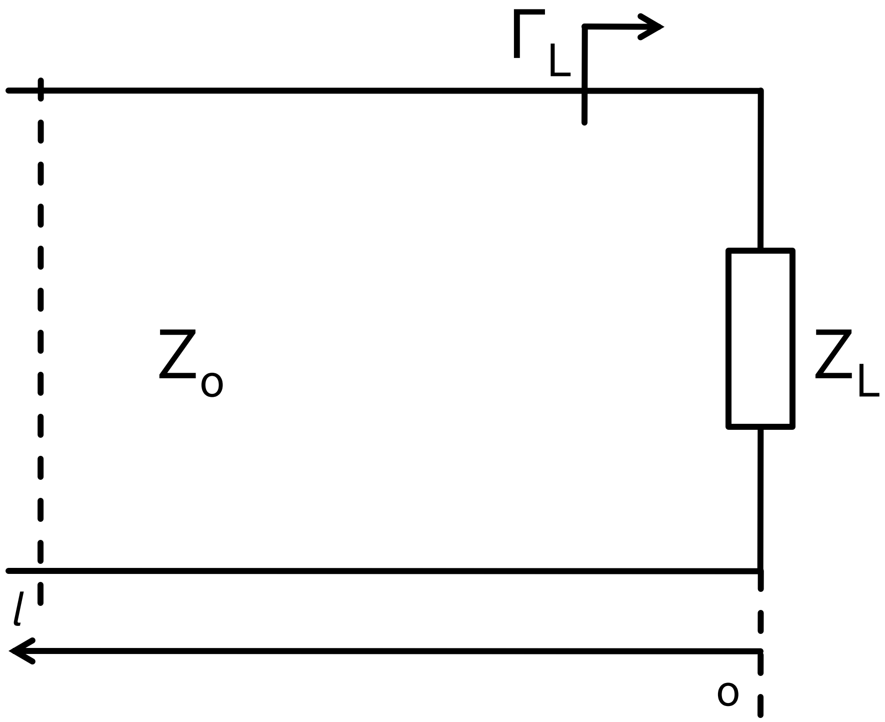
$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$\lambda = \frac{2\pi}{\omega\cdot\sqrt{LC}}$$

$$v_f = \frac{1}{\sqrt{LC}}$$

The lossless line



$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \qquad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

- Z_o real

The lossless line

$$V(z) = V_0^+ \cdot \left(e^{-j\cdot\beta\cdot z} + \Gamma \cdot e^{j\cdot\beta\cdot z}\right) \qquad I(z) = \frac{V_0^+}{Z_0} \cdot \left(e^{-j\cdot\beta\cdot z} - \Gamma \cdot e^{j\cdot\beta\cdot z}\right)$$

- time-average Power flow along the line

$$P_{avg} = \frac{1}{2} \cdot \text{Re}\left\{V(z) \cdot I(z)^*\right\} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \text{Re}\left\{1 - \Gamma^* \cdot e^{-2j\cdot\beta\cdot z} + \Gamma \cdot e^{2j\cdot\beta\cdot z} - |\Gamma|^2\right\}$$

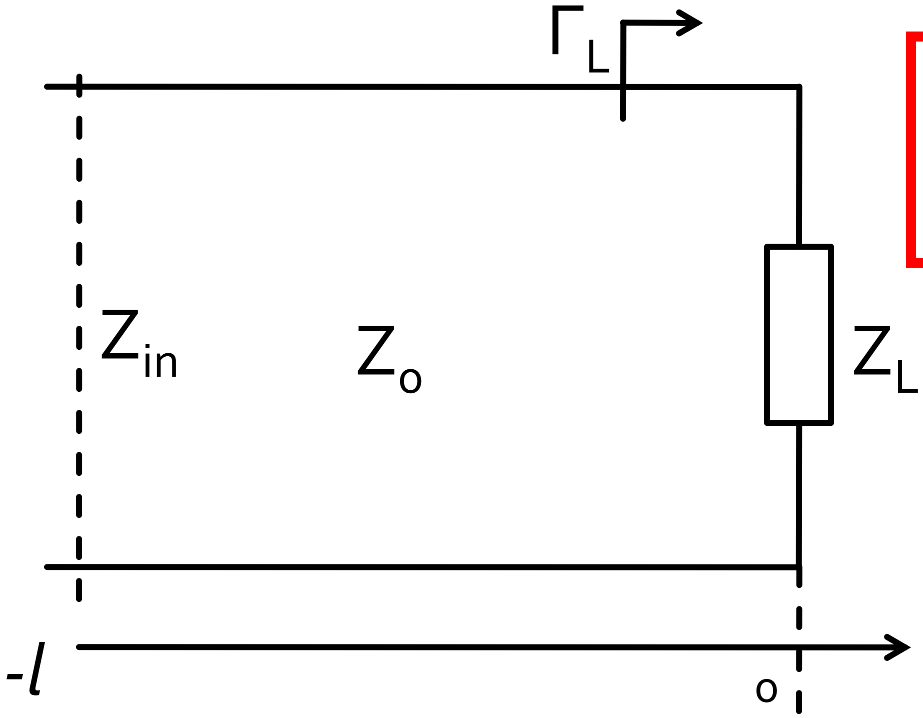
$$\left(z - z^*\right) = \text{Im}$$

$$P_{avg} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \left(1 - |\Gamma|^2\right)$$

- Total power delivered to the load = Incident power – "Reflected" power
- Return "Loss" [dB] $\quad \text{RL} = -20 \cdot \log|\Gamma| \quad [\text{dB}]$

The lossless line

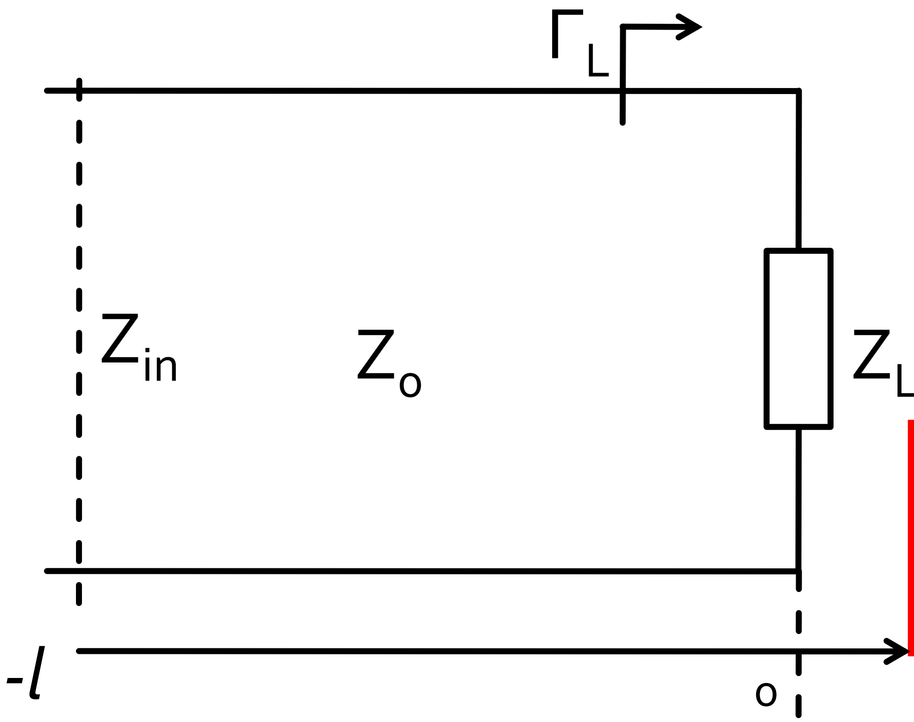
- input impedance of a length ***l*** of transmission line with characteristic impedance $\boldsymbol{Z_o}$, loaded with an arbitrary impedance $\boldsymbol{Z_L}$



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line

- input impedance is **frequency dependent** through $\beta \cdot l$



$$v_f = \frac{\omega}{\beta} = \lambda \cdot f \qquad \lambda = \frac{2\pi}{\beta}$$

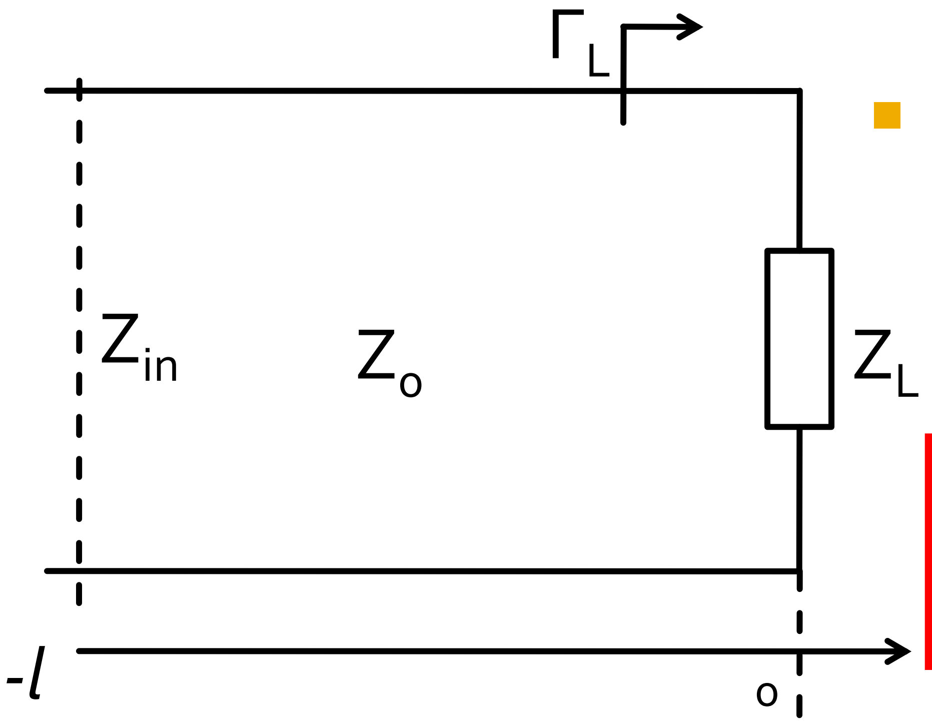
$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = \frac{2\pi \cdot f}{v_f} \cdot l = \frac{2\pi \cdot l}{v_f} \cdot f$$

frequency dependence is periodical, imposed by the tan trigonometric function

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

The lossless line, special cases

- $l = k \cdot \lambda/2$ $\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = k \cdot \pi$ $\tan \beta \cdot l = 0$ $$Z_{in} = Z_0$$
- $l = \lambda/4 + k \cdot \lambda/2$ $\tan \beta \cdot l \to \infty$ $$Z_{in} = \frac{Z_0^{\,2}}{Z_L}$$



- quarter-wave transformer

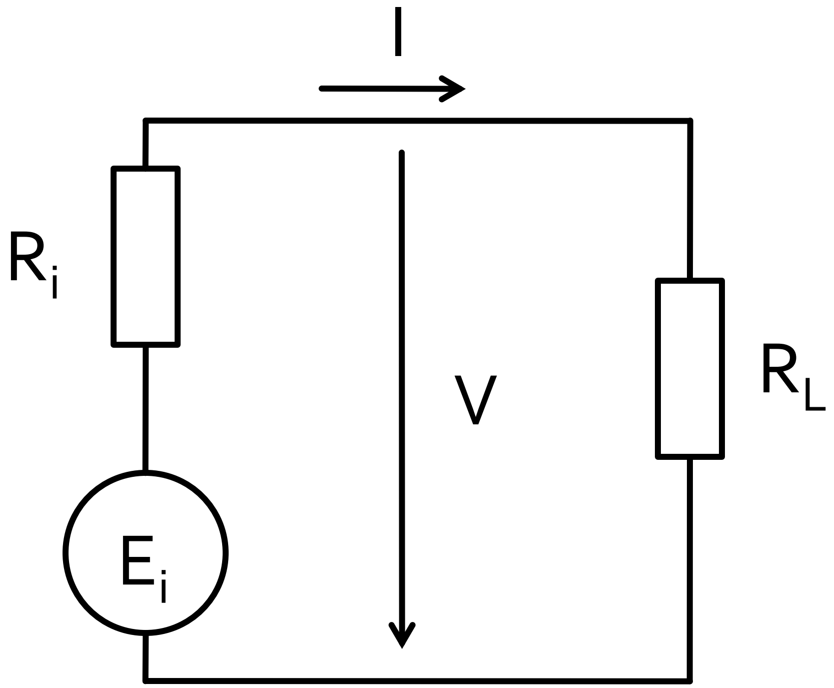
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Impedance Matching with Impedance Transformers (Lab 1)

Impedance Matching

Matching, real impedances

- Source matched to load



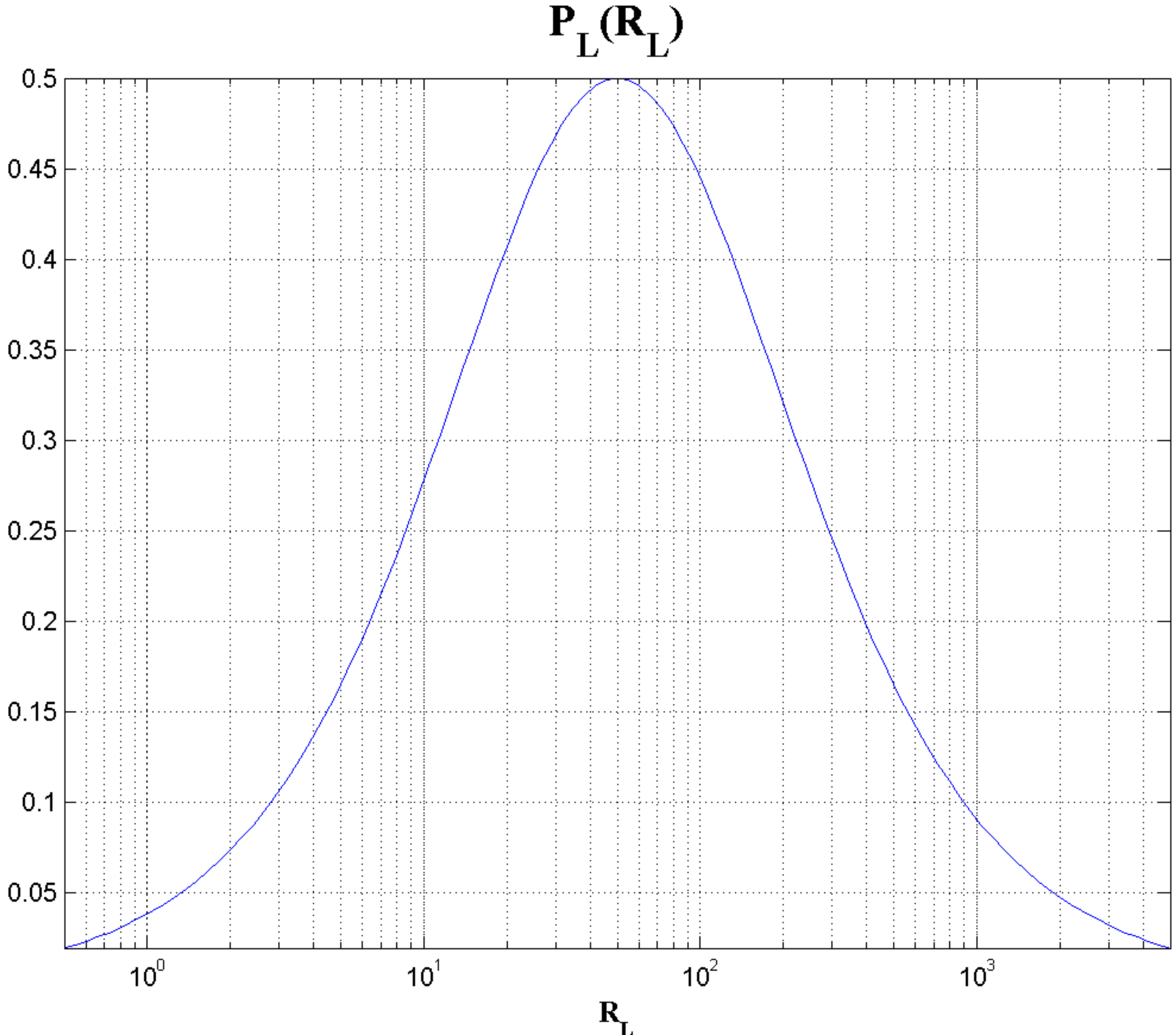
$$I = \frac{E_i}{R_i + R_L}$$

$$V = \frac{E_i \cdot R_L}{R_i + R_L}$$

$$P_L = R_L \cdot I^2$$

$$P_L = \frac{R_L \cdot E_i^2}{(R_i + R_L)^2}$$

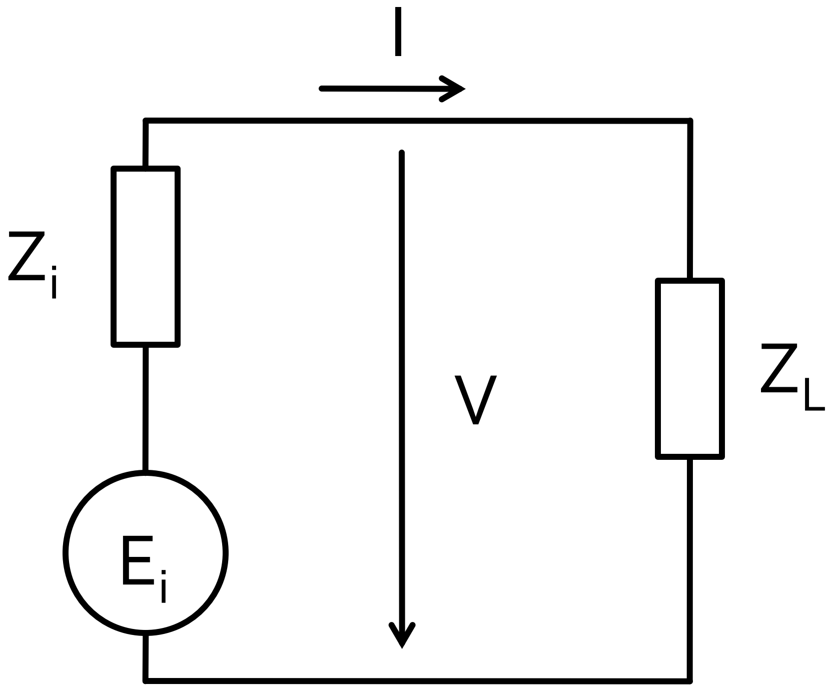
Matching, real impedances



$$R_L = R_i$$

Matching, complex impedances

- Source matched to load



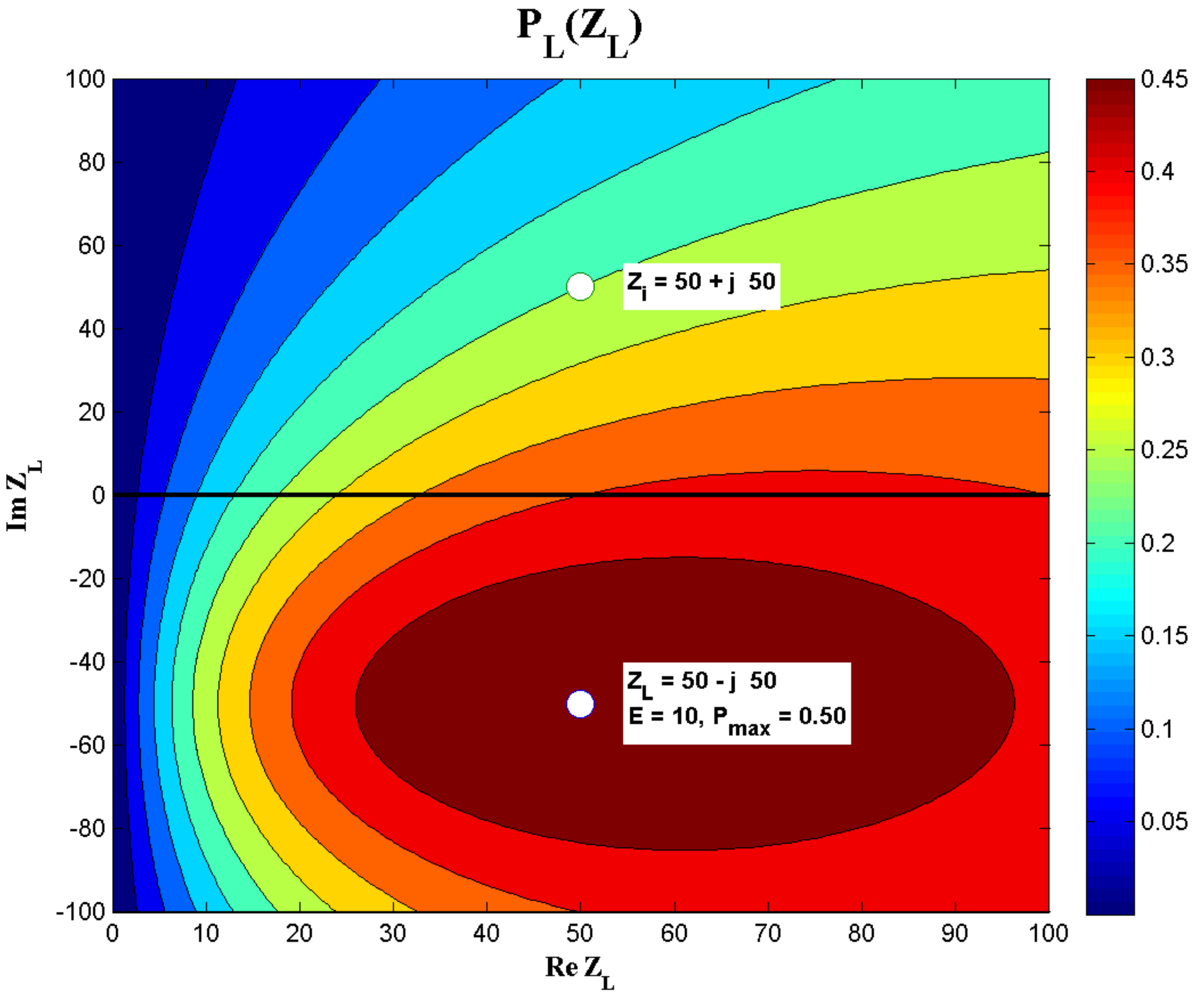
$$I = \frac{E_i}{Z_i + Z_L}$$

$$V = \frac{E_i \cdot Z_L}{Z_i + Z_L}$$

$$P_L = \mathrm{Re}\left\{ Z_L \cdot \left| I \right|^2 \right\}$$

$$P_L = \mathrm{Re}\left\{ Z_L \right\} \cdot \left| \frac{E_i}{Z_i + Z_L} \right|^2$$

Matching, example



$$Z_L = Z_i^*$$

Matching , from the point of view of power transmission

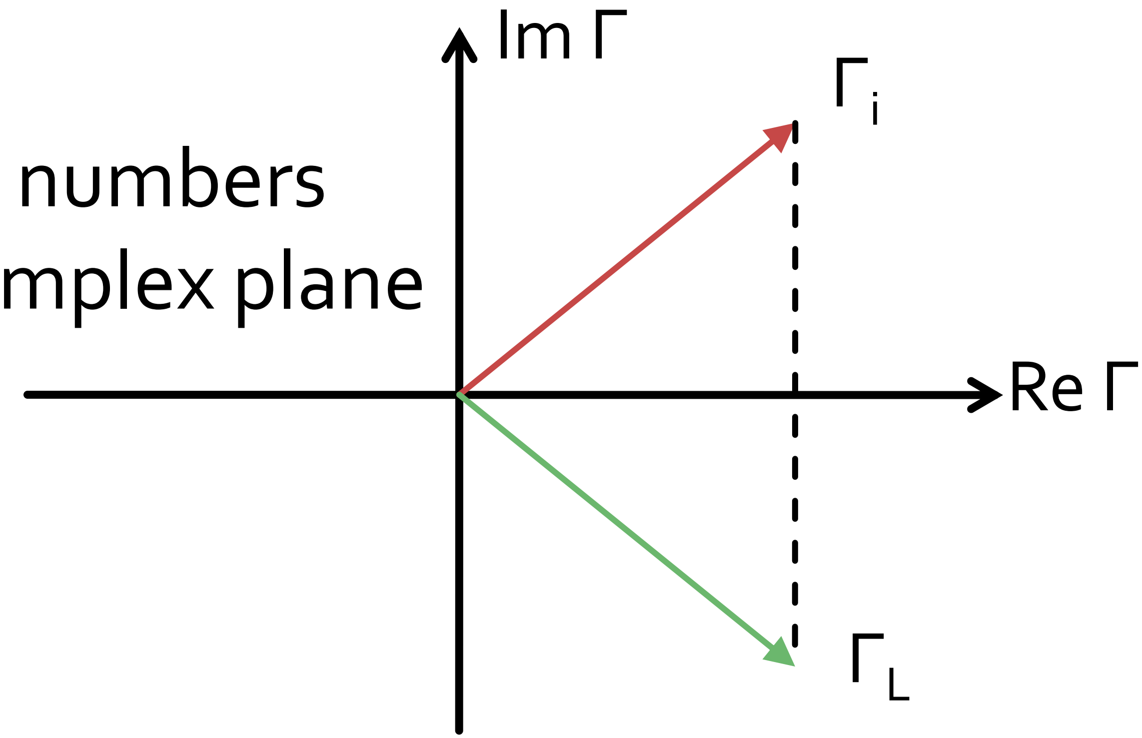
$$Z_L = Z_i^*$$

If we choose a real Zo

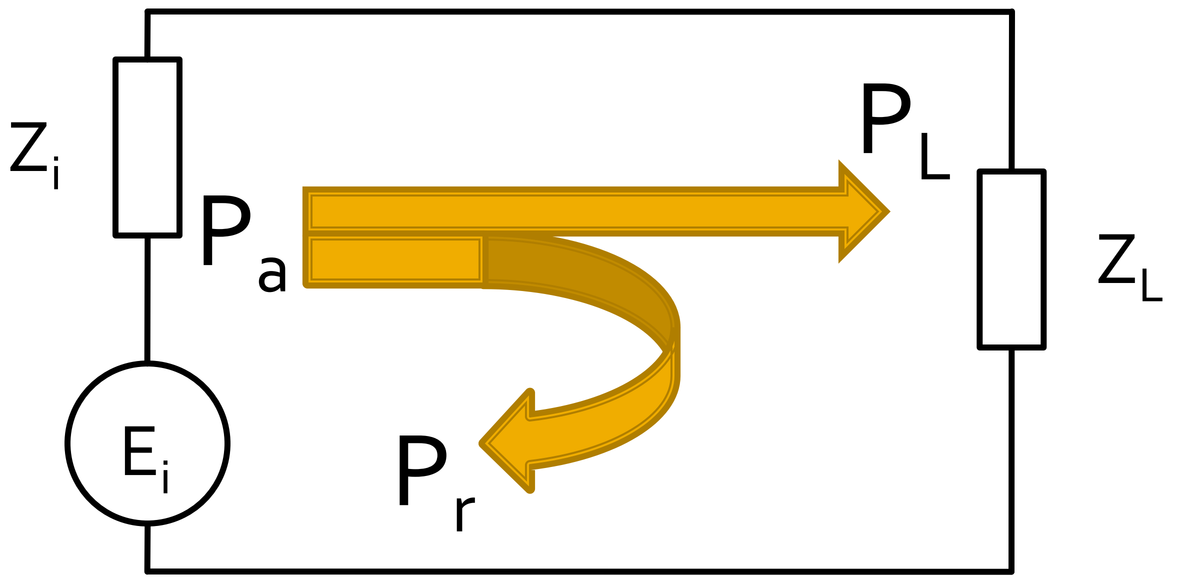
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

- complex numbers
- in the complex plane



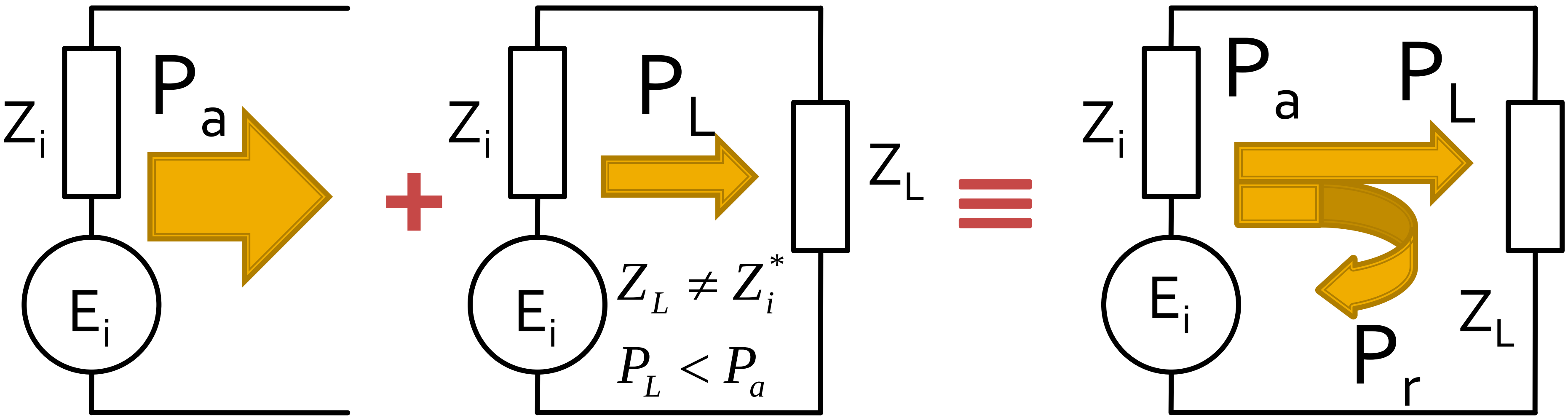
Reflection and power / Model



$$Z_L \neq Z_i^*$$

- ~~Power reflection~~
- Power of the reflected wave

Reflection and power / Model



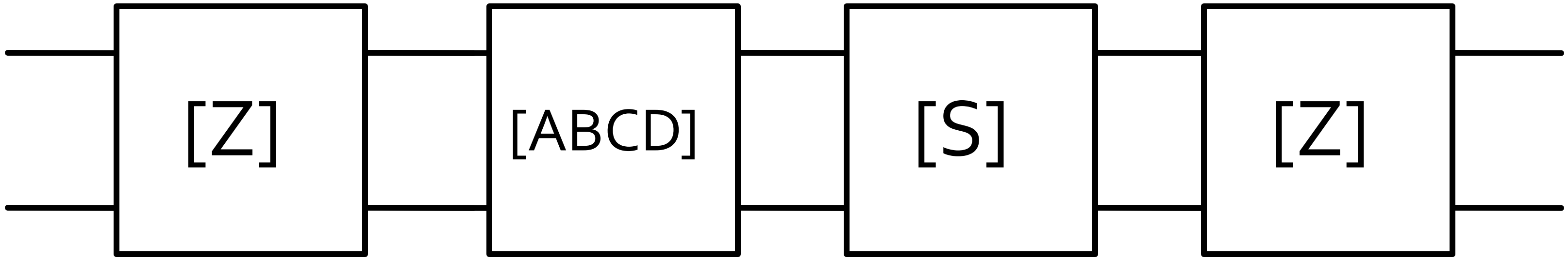
- The source has the ability to sent to the load a certain maximum power (available power) P_a
- For a particular load the power sent to the load is less than the maximum (mismatch) $P_L < P_a$
- The phenomenon is **"as if"** (model) some of the power is reflected $P_r = P_a - P_L$
- The power is a **scalar !**

Lecture 3

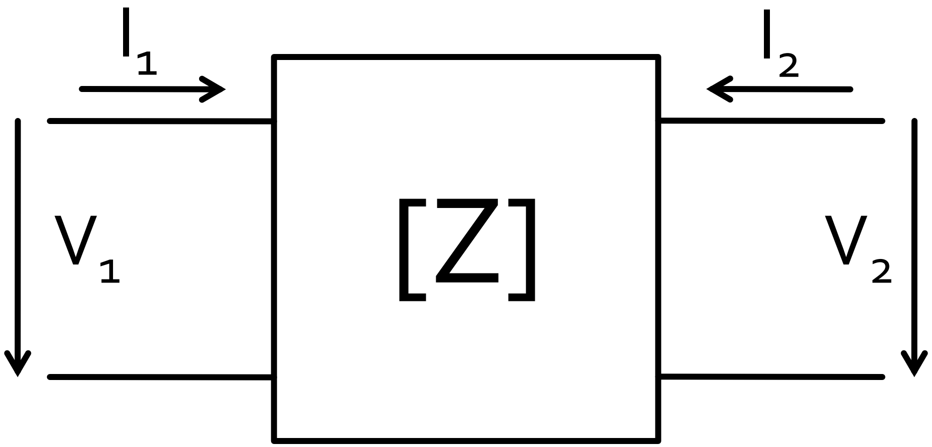
Microwave Network Analysis

Network Analysis

- We try to separate a complex circuit into individual blocks
- These are analyzed separately (decoupled from the rest of the circuit) and are characterized only by the port level signals (**black box**)
- Network-level analysis allows you to put together individual block results and get a total result for the entire circuit



Impedance matrix – Z



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

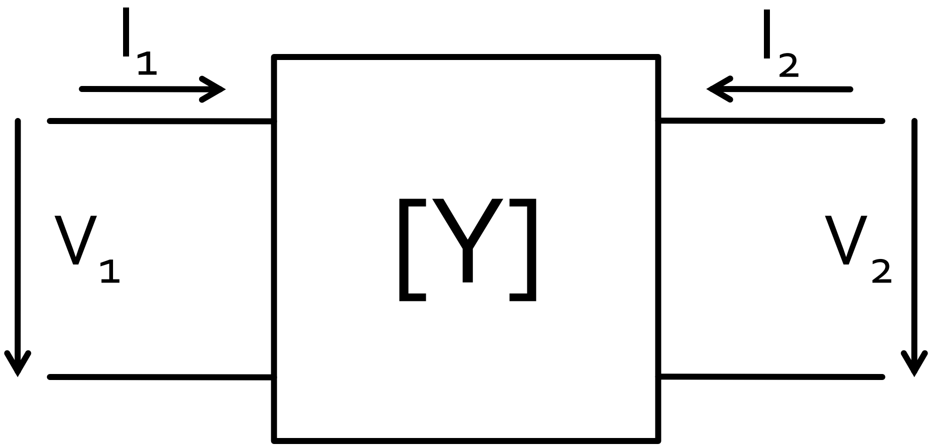
$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2$$
$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2$$

$$V_1 = Z_{11} \cdot I_1 \Big|_{I_2=0} \qquad Z_{11} = \frac{V_1}{I_1}\Bigg|_{I_2=0}$$

- Z11 – input impedance with open-circuited output

$$Z_{11} = \frac{V_1}{I_1}\Bigg|_{I_2=0} \qquad Z_{12} = \frac{V_1}{I_2}\Bigg|_{I_1=0} \qquad Z_{21} = \frac{V_2}{I_1}\Bigg|_{I_2=0} \qquad Z_{22} = \frac{V_2}{I_2}\Bigg|_{I_1=0}$$

Admittance matrix – Y



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

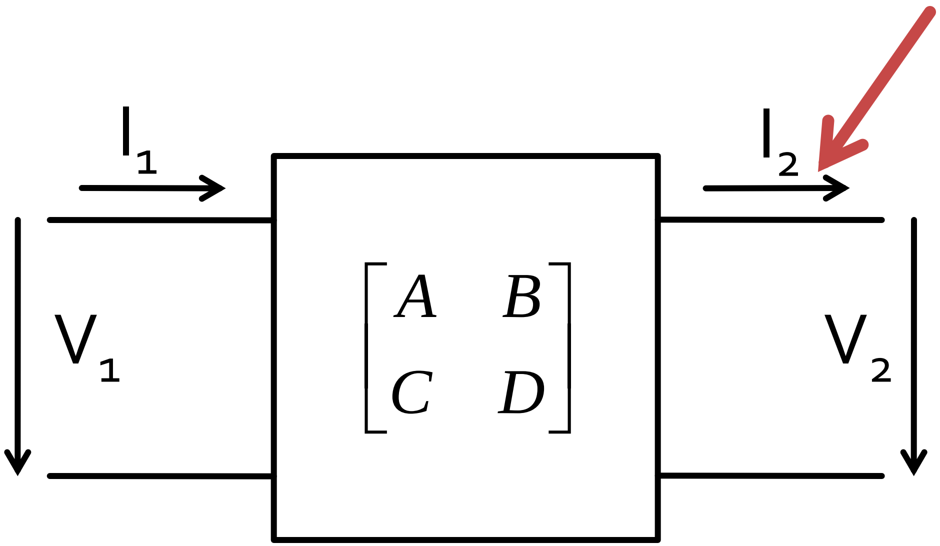
$$I_1 = Y_{11} \cdot V_1 + Y_{12} \cdot V_2$$
$$I_2 = Y_{21} \cdot V_1 + Y_{22} \cdot V_2$$

$$I_1 = Y_{11} \cdot V_1 \big|_{V_2=0} \qquad Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

- Y11 – input admittance with short-circuited output

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \qquad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

ABCD (transmission) matrix



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

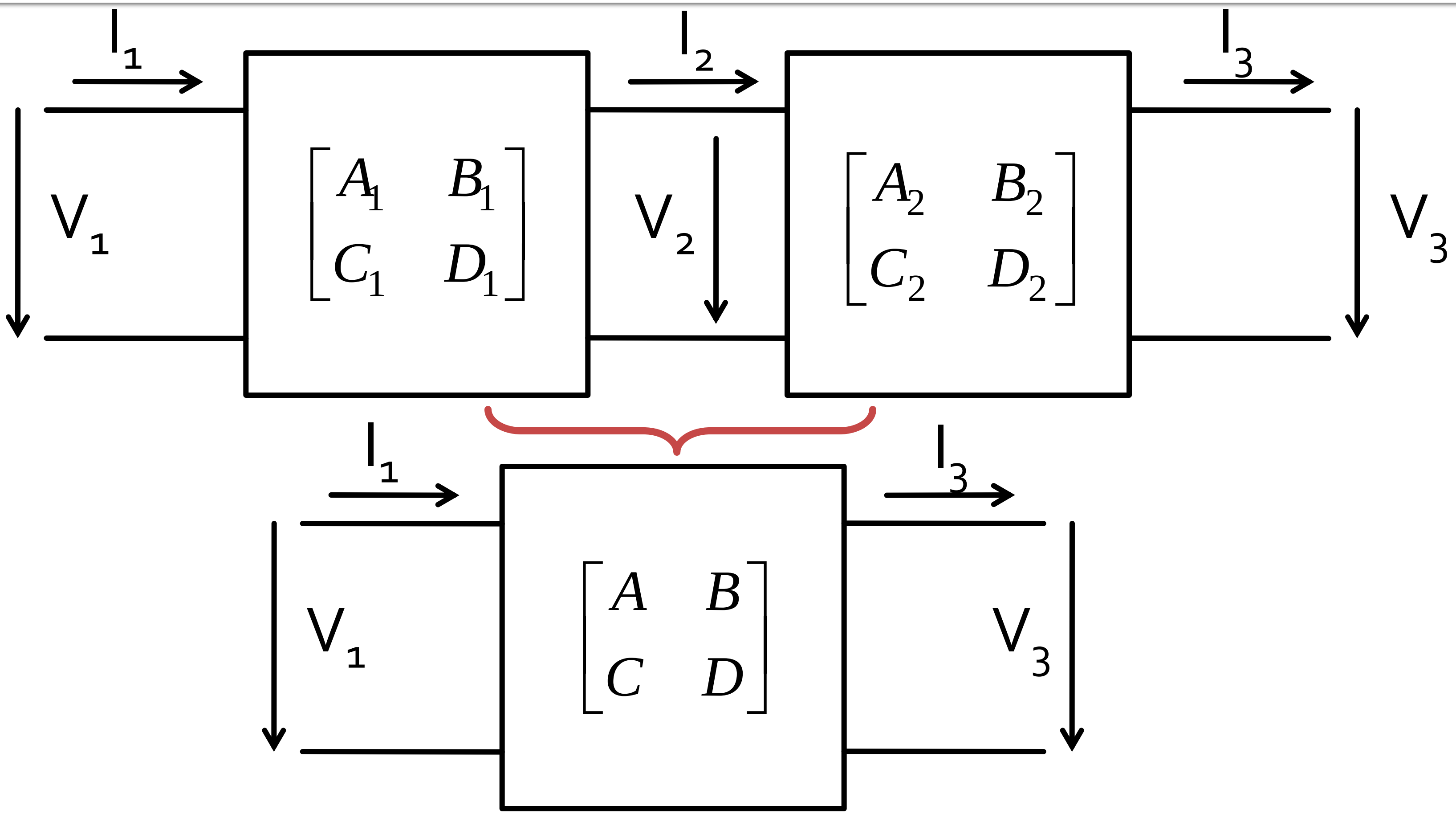
$$V_1 = A \cdot V_2 + B \cdot I_2$$

$$I_1 = C \cdot V_2 + D \cdot I_2$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \frac{1}{A \cdot D - B \cdot C} \cdot \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} \qquad B = \left. \frac{V_1}{I_2} \right|_{V_2=0} \qquad C = \left. \frac{I_1}{V_2} \right|_{I_2=0} \qquad D = \left. \frac{I_1}{I_2} \right|_{V_2=0}$$

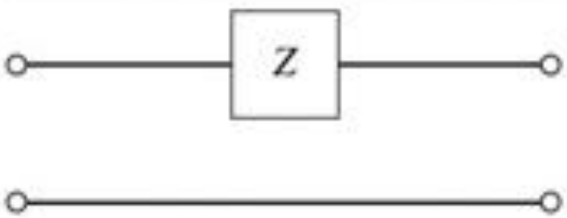
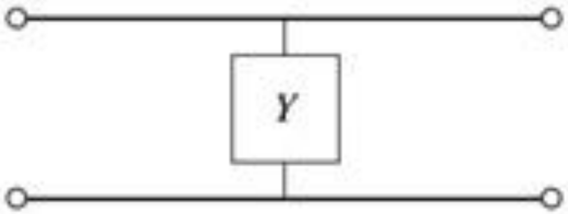
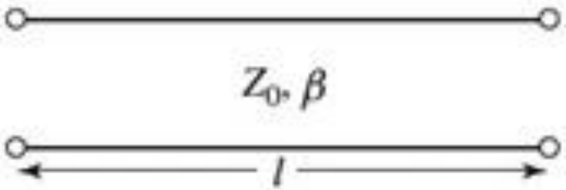
ABCD (transmission) matrix



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

Library of ABCD matrices

TABLE 4.1 ***ABCD* Parameters of Some Useful Two-Port Circuits**

Circuit	*ABCD* Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos \beta\ell$ $C = jY_0 \sin \beta\ell$	$B = jZ_0 \sin \beta\ell$ $D = \cos \beta\ell$

Library of ABCD matrices

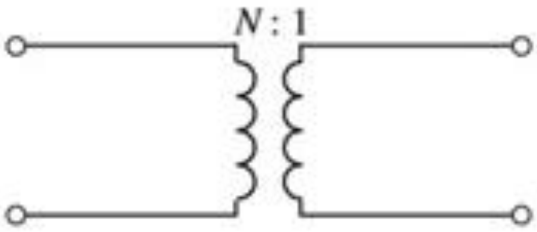
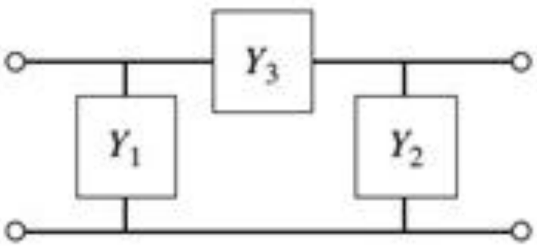
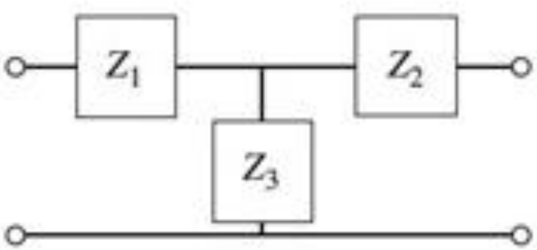
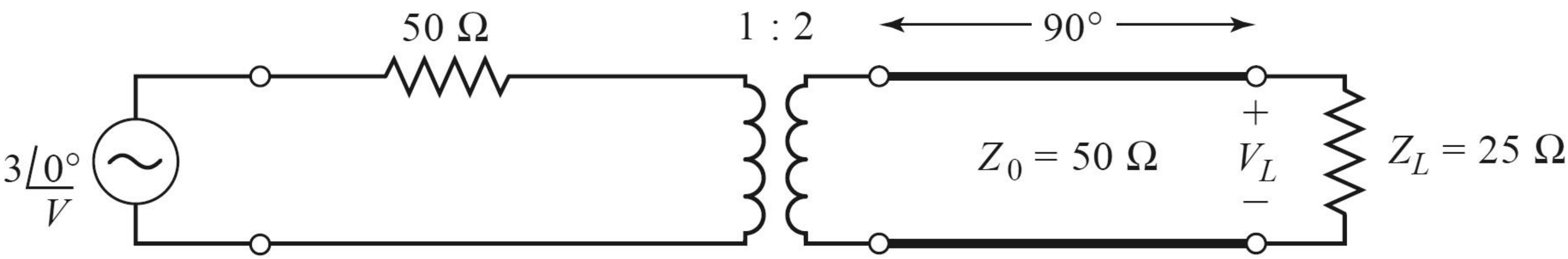
	$A = N$ $C = 0$	$B = 0$ $D = \frac{1}{N}$
	$A = 1 + \frac{Y_2}{Y_3}$ $C = Y_1 + Y_2 + \frac{Y_1 Y_2}{Y_3}$	$B = \frac{1}{Y_3}$ $D = 1 + \frac{Y_1}{Y_3}$
	$A = 1 + \frac{Z_1}{Z_3}$ $C = \frac{1}{Z_3}$	$B = Z_1 + Z_2 + \frac{Z_1 Z_2}{Z_3}$ $D = 1 + \frac{Z_2}{Z_3}$

Table 4.1

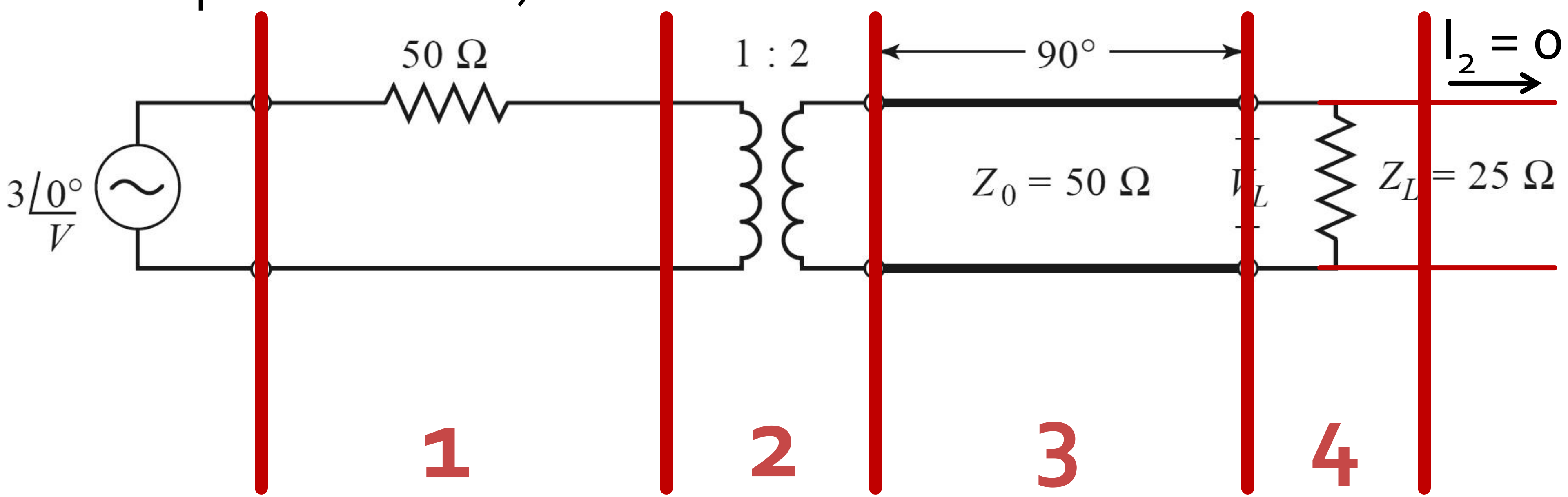
Example for ABCD matrix

- Find the voltage V_L across the load resistor in the circuit shown below



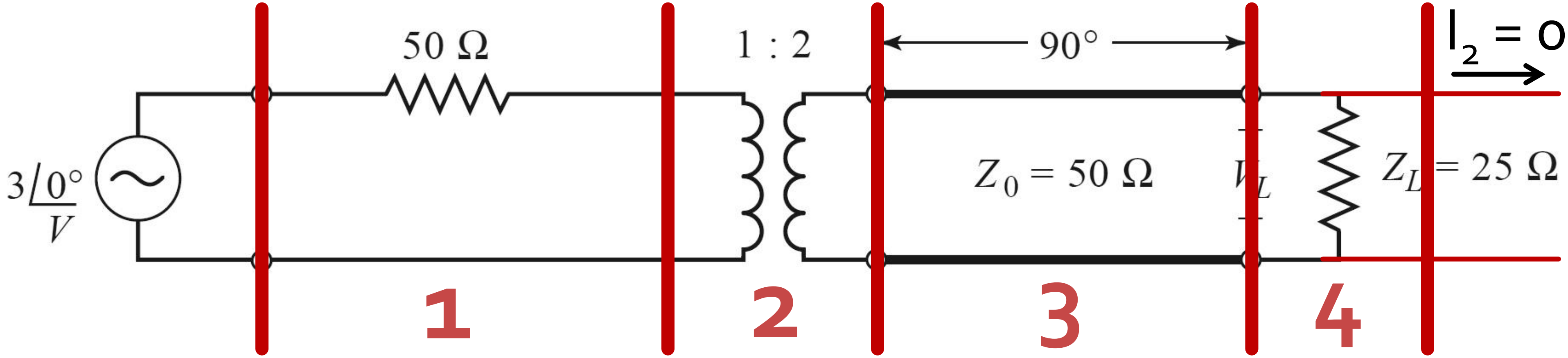
Example for ABCD matrix

- We break the circuit in elementary sections
- Sources are left outside
- If necessary, input and output ports are created (and left open-circuited)



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = M_1 \cdot M_2 \cdot M_3 \cdot M_4 \quad V_1 = A \cdot V_2 + B \cdot I_2\Big|_{I_2=0} \quad V = A \cdot V_L \rightarrow V_L = \frac{V}{A}$$

Example for ABCD matrix



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 50 \cdot j \\ \frac{j}{50} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ \frac{1}{25} & 1 \end{bmatrix} = \begin{bmatrix} 3 \cdot j & 25 \cdot j \\ \frac{j}{25} & 0 \end{bmatrix}$$

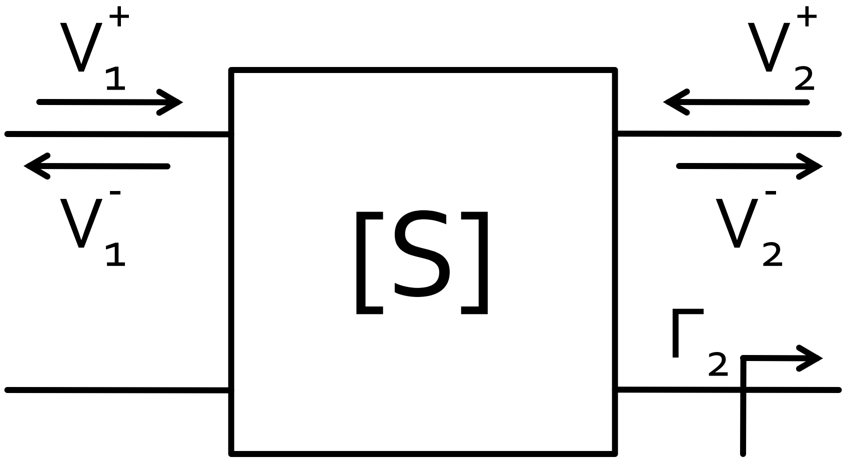
$$V_L = \frac{V}{A} = \frac{3\angle 0^\circ}{3 \cdot j} = 1\angle 90^\circ$$

Continued

Microwave Network Analysis

Scattering matrix – S

- Scattering parameters



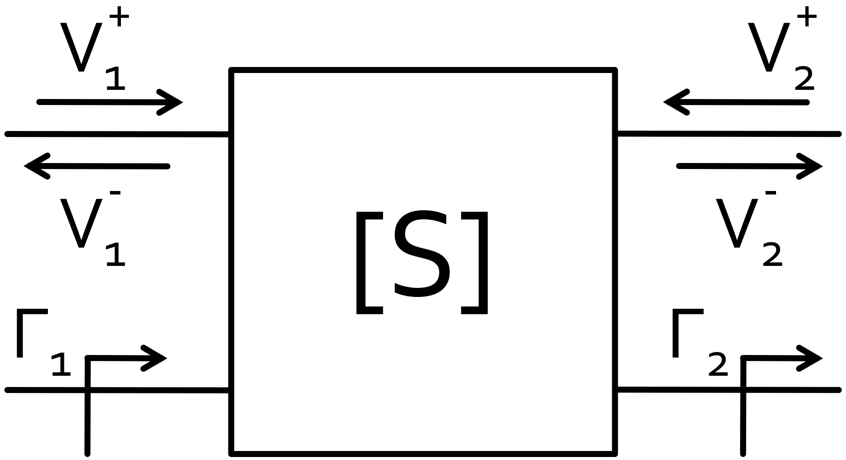
$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} \qquad S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$

- $V_2^+ = 0$ meaning: port 2 is terminated in matched load to avoid reflections towards the port

$$\Gamma_2 = 0 \rightarrow V_2^+ = 0$$

Scattering matrix – S



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} = \left. \Gamma_1 \right|_{\Gamma_2=0}$$

$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0} = \left. T_{21} \right|_{\Gamma_2=0}$$

- S11 is the reflection coefficient seen looking into port *1* when port *2* is terminated in matched load
- S21 is the transmission coefficient from port *1* (**second** index) to port *2* (**first** index) when port *2* is terminated in matched load

Scattering matrix – S

- S matrix can be extended to multiple ports

$$S_{ii} = \left.\frac{V_i^-}{V_i^+}\right|_{V_k^+=0,\forall k\neq i} \qquad S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{V_k^+=0,\forall k\neq j}$$

  - S_{ii} is the reflection coefficient seen looking into port ***i*** when all other ports are terminated in matched loads
  - S_{ij} is the transmission coefficient from port ***j*** (**second** index) to port ***i*** (**first** index) when all other ports are terminated in matched loads

Properties of S matrix

- If port i is connected to a transmission line with charateristic impedance Z_{0i}

$$[Z_0]=\begin{bmatrix} Z_{01} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Z_{0n} \end{bmatrix}$$

- Lecture 2

$$V(z)=V_0^+ e^{-j\cdot\beta\cdot z}+V_0^- e^{j\cdot\beta\cdot z} \qquad I(z)=\frac{V_0^+}{Z_0}e^{-j\cdot\beta\cdot z}-\frac{V_0^-}{Z_0}e^{j\cdot\beta\cdot z}$$

In the port's reference plane, z=0

$$V_i=V_i^+ + V_i^- \qquad I_i=\frac{V_i^+}{Z_{0i}}-\frac{V_i^-}{Z_{0i}}$$

- Relation to Z matrix

$$[Z]\cdot[I]=[V]$$

$$[Z]\cdot[I]=[Z_0]^{-1}\cdot[Z]\cdot[V^+]-[Z_0]^{-1}\cdot[Z]\cdot[V^-] \qquad [V]=[V^+]+[V^-]$$

$$[Z_0]^{-1}\cdot[Z]\cdot[V^+]-[Z_0]^{-1}\cdot[Z]\cdot[V^-]=[V^+]+[V^-] \qquad ([Z]-[Z_0])\cdot[V^+]=([Z]+[Z_0])\cdot[V^-]$$

$$[V^-]=[S]\cdot[V^+] \qquad \boxed{[S]=([Z]-[Z_0])\cdot([Z]+[Z_0])^{-1}}$$

A Shift in Reference Planes

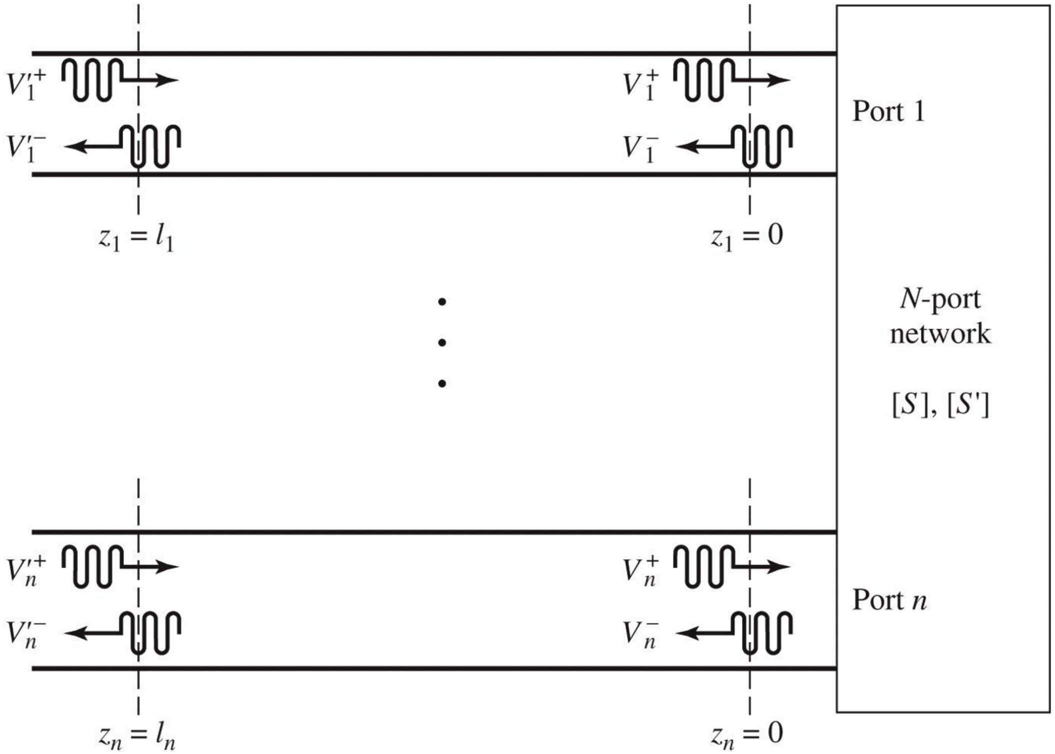


Figure 4.9

$$[S'] = \begin{bmatrix} e^{-j\cdot\theta_1} & 0 & \cdots & 0 \\ 0 & e^{-j\cdot\theta_2} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & e^{-j\cdot\theta_N} \end{bmatrix} \cdot [S] \cdot \begin{bmatrix} e^{-j\cdot\theta_1} & 0 & \cdots & 0 \\ 0 & e^{-j\cdot\theta_2} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & e^{-j\cdot\theta_N} \end{bmatrix}$$

Properties of S matrix (Z,Y)

- Reciprocal networks (no active circuits, no ferrites)

$$Z_{ij} = Z_{ji}, \forall j \neq i$$

$$Y_{ij} = Y_{ji}, \forall j \neq i$$

$$S_{ij} = S_{ji}, \forall j \neq i$$

$$[S] = [S]^t$$

- Lossless networks

$$\mathrm{Re}\{Z_{ij}\} = 0, \forall i, j$$

$$\mathrm{Re}\{Y_{ij}\} = 0, \forall i, j$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{kj}^{*} = \delta_{ij}, \forall i, j$$

$$[S]^{*} \cdot [S]^{t} = [1]$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{ki}^{*} = 1$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{kj}^{*} = 0, \forall i \neq j$$

Generalized Scattering Parameters

- The total voltage and current on a transmission line in terms of the incident and reflected voltage wave amplitudes

$$V = V_0^+ + V_0^- \qquad I = \frac{1}{Z_0} \cdot \left(V_0^+ - V_0^-\right)$$

In the port's reference plane, z=0

- We find the incident and reflected voltage wave amplitudes

$$V_0^+ = \frac{V + Z_0 \cdot I}{2} \qquad V_0^- = \frac{V - Z_0 \cdot I}{2}$$

- The average power delivered to a load :

$$P_L = \frac{1}{2} \cdot \mathrm{Re}\left\{V \cdot I^*\right\} = \frac{1}{2 \cdot Z_0} \cdot \mathrm{Re}\left\{\left|V_0^+\right|^2 - V_0^+ \cdot V_0^{-*} + V_0^{+*} \cdot V_0^- - \left|V_0^-\right|^2\right\}$$

$$\left(z - z^*\right) = \mathrm{Im}$$

$$P_L = \frac{1}{2 \cdot Z_0} \cdot \left(\left|V_0^+\right|^2 - \left|V_0^-\right|^2\right)$$

Generalized Scattering Parameters

- We define the power wave amplitudes a and b

$$a = \frac{V + Z_R \cdot I}{2 \cdot \sqrt{R_R}}$$ the incident power wave

$$Z_R = R_R + j \cdot X_R$$

Any complex impedance, named reference impedance

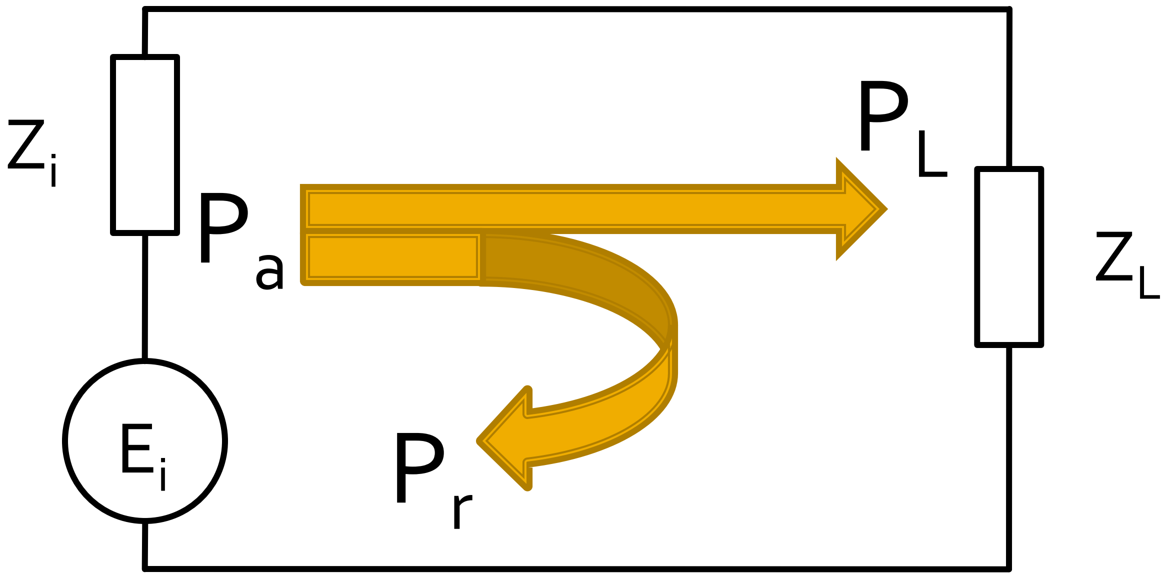
$$b = \frac{V - Z_R^* \cdot I}{2 \cdot \sqrt{R_R}}$$ the reflected power wave

- Total voltage and current in terms of the power wave amplitudes

$$V = \frac{Z_R^* \cdot a + Z_R \cdot b}{\sqrt{R_R}}$$

$$I = \frac{a - b}{\sqrt{R_R}}$$

Reflection and power / Model – L3



$$P_a = \frac{|E_i|^2}{4R_i}$$

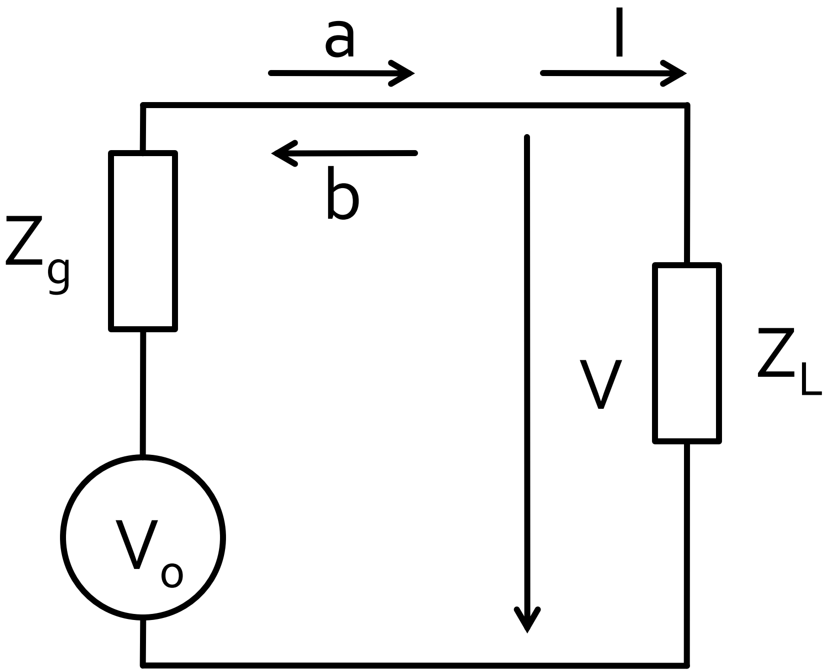
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0}$$

$$P_r = \frac{|E_i|^2}{4R_i} \cdot \left[\frac{(R_i - R_L)^2 + (X_i + X_L)^2}{(R_i + R_L)^2 + (X_i + X_L)^2} \right] = P_a \cdot |\Gamma|^2$$

- Γ, power reflection coefficient

Power waves



$$P_L = \frac{1}{2} \cdot \mathrm{Re}\left\{V \cdot I^*\right\}$$

$$P_L = \frac{1}{2} \cdot \mathrm{Re}\left\{\frac{Z_R^* \cdot a + Z_R \cdot b}{\sqrt{R_R}} \cdot \left(\frac{a-b}{\sqrt{R_R}}\right)^*\right\}$$

$$P_L = \frac{1}{2R_R} \cdot \mathrm{Re}\left\{Z_R^* \cdot |a|^2 - Z_R^* \cdot a \cdot b^* + Z_R \cdot a^* \cdot b - Z_R \cdot |b|^2\right\}$$

$$P_L = \frac{1}{2} \cdot |a|^2 - \frac{1}{2} \cdot |b|^2$$

$$\Gamma_p = \frac{b}{a} = \frac{V - Z_R^* \cdot I}{V + Z_R \cdot I} = \frac{Z_L - Z_R^*}{Z_L + Z_R}$$

Power waves

$$V = \frac{V_0 \cdot Z_L}{Z_g + Z_L} \qquad I = \frac{V_0}{Z_g + Z_L} \qquad P_L = \frac{V_0^2}{2} \cdot \frac{R_L}{\left| Z_g + Z_L \right|^2}$$

- If we choose $Z_R = Z_L^*$

$$a = \frac{V + Z_R \cdot I}{2 \cdot \sqrt{R_R}} = V_0 \cdot \frac{\dfrac{Z_L}{Z_g + Z_L} + \dfrac{Z_L^*}{Z_g + Z_L}}{2 \cdot \sqrt{R_L}} = V_0 \cdot \frac{\sqrt{R_L}}{Z_g + Z_L}$$

$$b = \frac{V - Z_R^* \cdot I}{2 \cdot \sqrt{R_R}} = V_0 \cdot \frac{\dfrac{Z_L}{Z_g + Z_L} - \dfrac{Z_L}{Z_g + Z_L}}{2 \cdot \sqrt{R_L}} = 0$$

$$P_L = \frac{1}{2} \cdot \left| a \right|^2 = \frac{V_0^2}{2} \cdot \frac{R_L}{\left| Z_g + Z_L \right|^2}$$

Power waves

- When the load is conjugately matched to the generator

$$Z_g = Z_L^* \qquad P_{L\max} = \frac{1}{2}\cdot|a|^2 = \frac{V_0^2}{8\cdot R_L}$$

- Power reflection: L3

$$Z_L = Z_i^* \qquad P_{L\max} \equiv P_a \qquad \Gamma = \frac{Z - Z_0^*}{Z + Z_0}$$

$$Z_L \neq Z_i^* \qquad P_r = P_a \cdot |\Gamma|^2 \qquad P_L = P_a - P_r = P_a - P_a \cdot |\Gamma|^2 = P_a \cdot \left(1 - |\Gamma|^2\right)$$

- Power reflection: L3

$$P_{L\max} \equiv P_a = \frac{1}{2}\cdot|a|^2 \qquad P_L = \frac{1}{2}\cdot|a|^2 - \frac{1}{2}\cdot|b|^2 \qquad \Gamma_p = \frac{b}{a} = \frac{V - Z_R^*\cdot I}{V + Z_R\cdot I} = \frac{Z_L - Z_R^*}{Z_L + Z_R}$$

$$P_L = \frac{1}{2}\cdot|a|^2 - \frac{1}{2}\cdot|a|^2\cdot|\Gamma_p|^2 \qquad P_L = P_a\cdot\left(1 - |\Gamma_p|^2\right) \qquad P_r = P_a\cdot|\Gamma_p|^2 = \frac{1}{2}\cdot|b|^2$$

Power waves

- To define the scattering matrix for power waves for an N-port network

$$[Z_R] = \begin{bmatrix} Z_{R1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & Z_{Rn} \end{bmatrix} \qquad [F] = \begin{bmatrix} 1/2\sqrt{R_{R1}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/2\sqrt{R_{Rn}} \end{bmatrix}$$

$$[a] = [F]\cdot([V] + [Z_R]\cdot[I])$$

$$[b] = [F]\cdot\left([V] - [Z_R]^* \cdot [I]\right)$$

$$[z]\cdot[I] = [V]$$

Power waves for N ports

$$[b]=[F]\cdot\left([Z]-[Z_R]^*\right)\cdot\left([Z]+[Z_R]\right)^{-1}\cdot[F]^{-1}\cdot[a]$$

- The scattering matrix for power waves, $[S_p]$

$$[b]=[S_p]\cdot[a]$$

$$[S_p]=[F]\cdot\left([Z]-[Z_R]^*\right)\cdot\left([Z]+[Z_R]\right)^{-1}\cdot[F]^{-1}$$

- But: $[S]=\left([Z]-[Z_0]\right)\cdot\left([Z]+[Z_0]\right)^{-1}$

- Typically

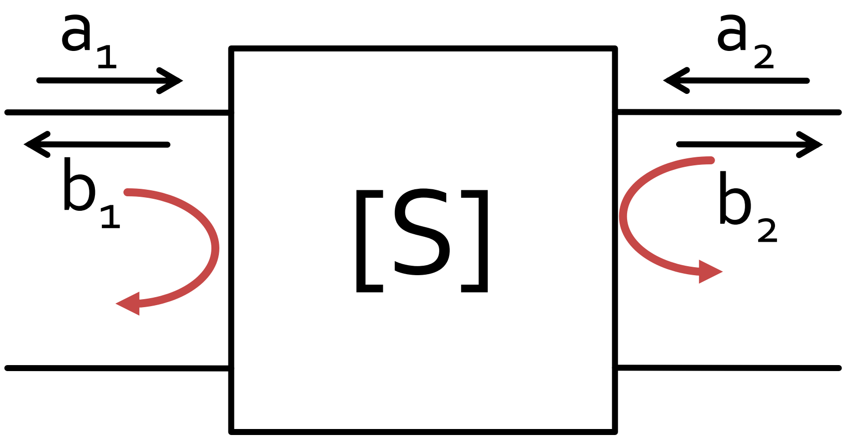
$$Z_{0i}=Z_{Ri}=R_0, \forall i$$

$$R_0=50\Omega$$

$$[S_p]\equiv[S]$$

- they coincide!!!

Scattering matrix – S

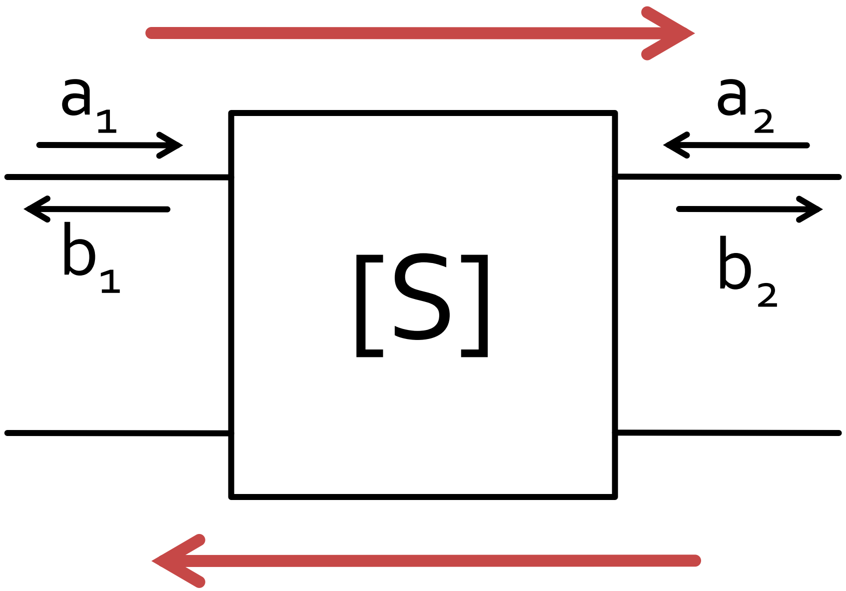


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

- S_{11} and S_{22} are reflection coefficients at ports 1 and 2 when the other port is matched

Scattering matrix – S

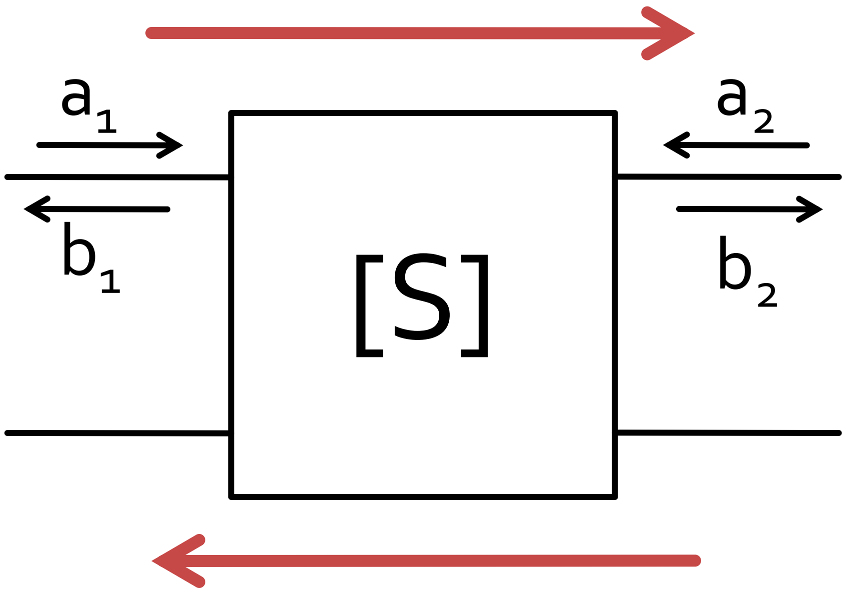


$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \qquad S_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0}$$

- S_{21} si S_{12} are signal amplitude gain when the other port is matched

Scattering matrix – S



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$|S_{21}|^2 = \frac{Power\ in\ Z_0\ load}{Power\ from\ Z_0\ source}$$

- a,b
  - information about signal power **AND** signal phase
- S_{ij}
  - network effect (gain) over signal power **including** phase information

Measuring S parameters - VNA

- Vector Network Analyzer

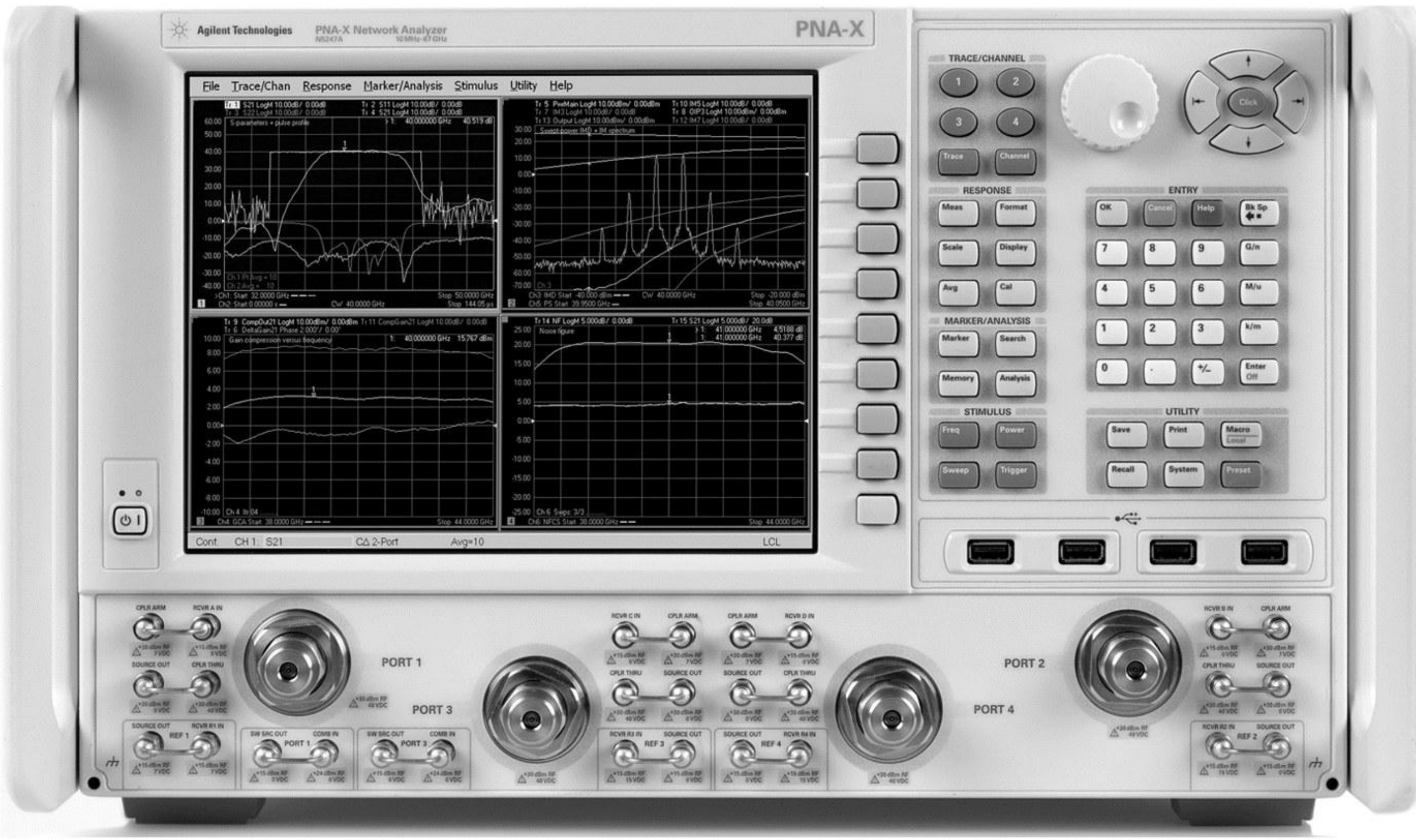


Figure 4.7
Courtesy of Agilent Technologies

Relation between two port S parameters and ABCD parameters

$$A = \sqrt{\frac{Z_{01}}{Z_{02}}}\frac{(1+S_{11}-S_{22}-\Delta S)}{2S_{21}}$$

$$B = \sqrt{Z_{01}Z_{02}}\frac{(1+S_{11}+S_{22}+\Delta S)}{2S_{21}}$$

$$C = \frac{1}{\sqrt{Z_{01}Z_{02}}}\frac{1-S_{11}-S_{22}+\Delta S}{2S_{21}}$$

$$D = \sqrt{\frac{Z_{02}}{Z_{01}}}\frac{1-S_{11}+S_{22}-\Delta S}{2S_{21}}$$

$$\Delta S = S_{11}S_{22} - S_{12}S_{21}$$

$$S_{11} = \frac{AZ_{02}+B-CZ_{01}Z_{02}-DZ_{01}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

$$S_{12} = \frac{2(AD-BC)\sqrt{Z_{01}Z_{02}}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

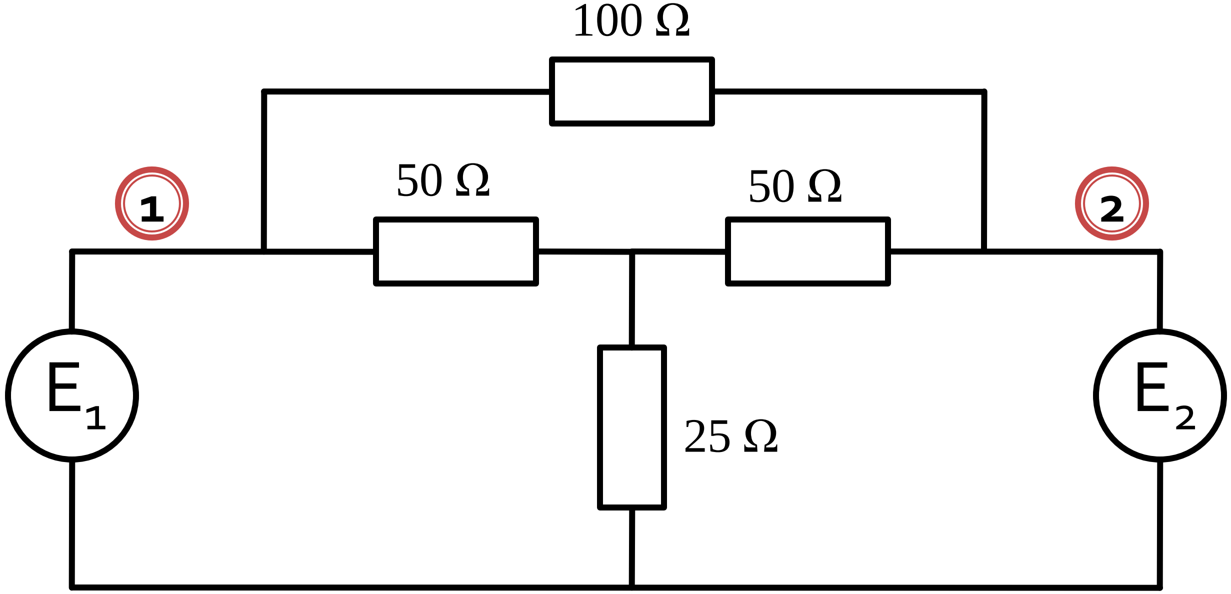
$$S_{21} = \frac{2\sqrt{Z_{01}Z_{02}}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

$$S_{22} = \frac{-AZ_{02}+B-CZ_{01}Z_{02}+DZ_{01}}{AZ_{02}+B+CZ_{01}Z_{02}+DZ_{01}}$$

Microwave Network Analysis

Even/Odd Mode Analysis

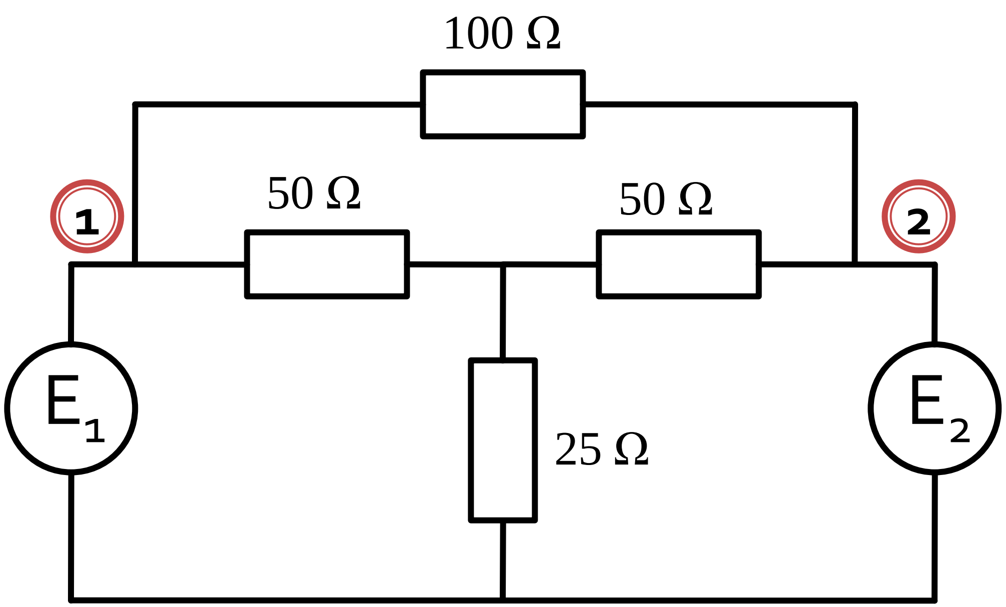
- useful method, necessary even for multiple ports
- example, resistors, two port circuitcircuit

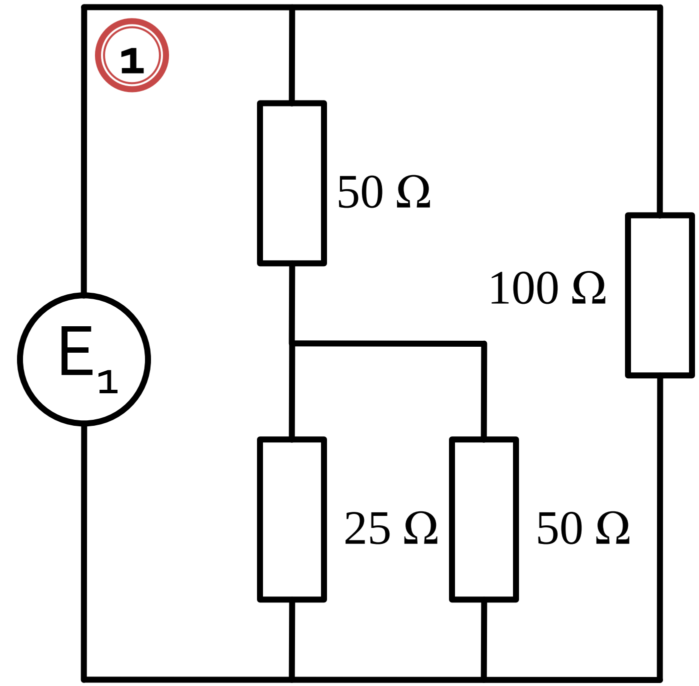


Even/Odd Mode Analysis

- assume we want to compute Y_{11}
- $E_2 = 0$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$



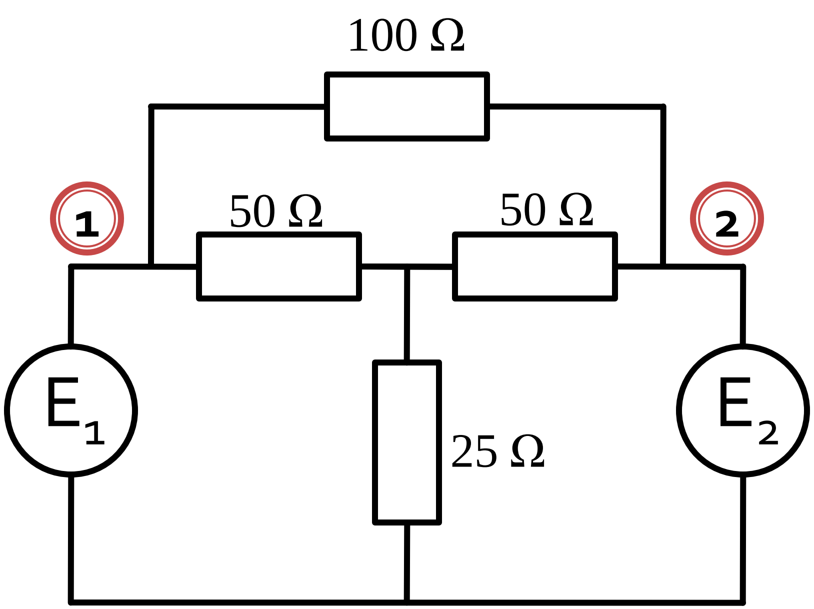


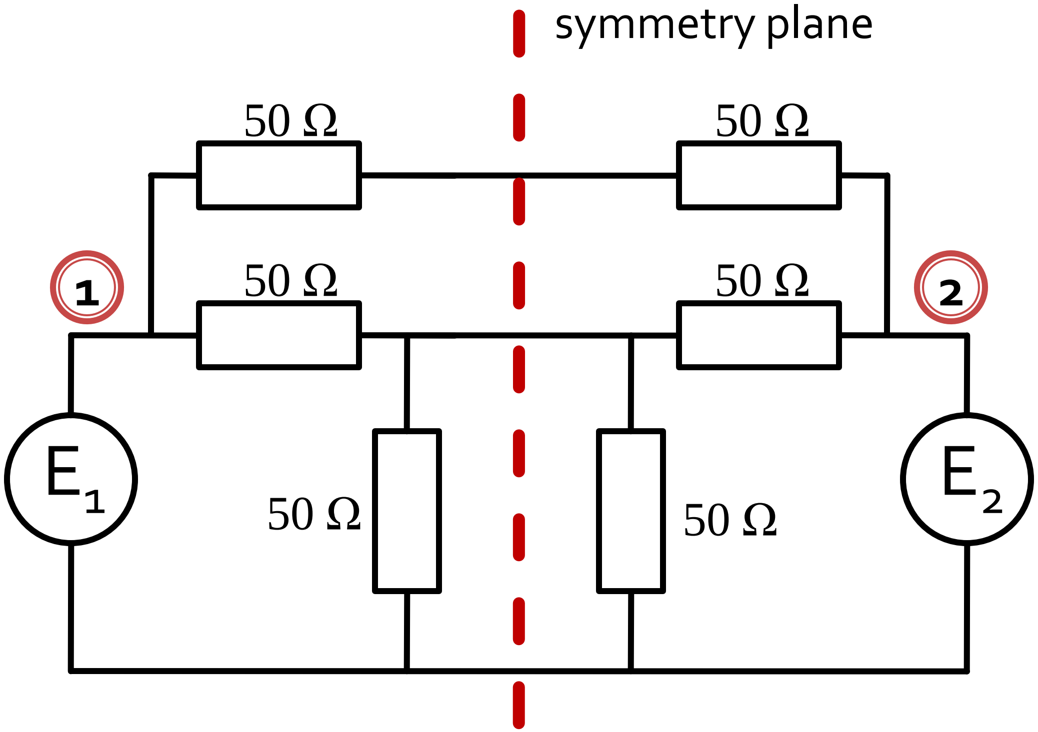
$$R_{ech} = 100\Omega \,||\, (50\Omega + 25\Omega \,||\, 50\Omega) =$$
$$= 100\Omega \,||\, (50\Omega + 16.67\Omega) = 100\Omega \,||\, 66.67\Omega = 40\Omega$$

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = 0.025S$$

Even/Odd Mode Analysis

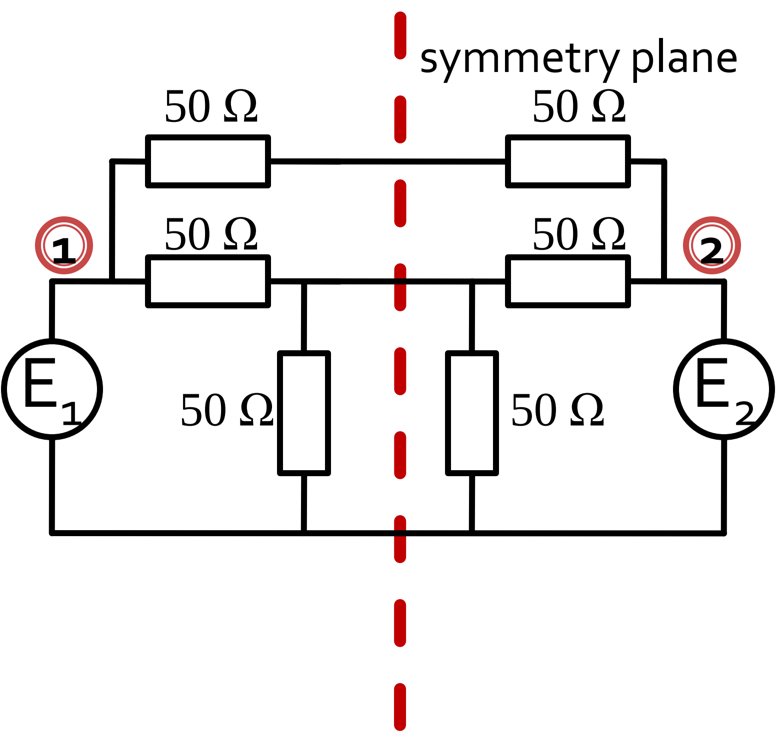
- Even/Odd mode analysis benefit from the existence of symmetry planes in the circuit
  - existing or
  - created (forced)

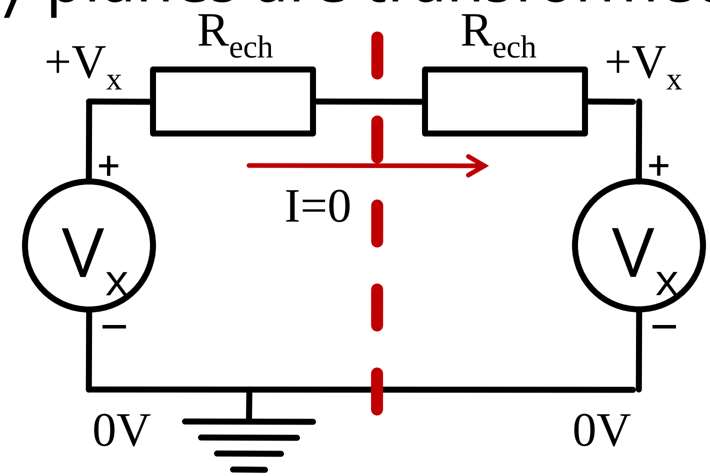




Even/Odd Mode Analysis

- when exciting the ports with symmetric/anti-symmetric sources the symmetry planes are transformed into:
  - open circuit
  - virtual ground

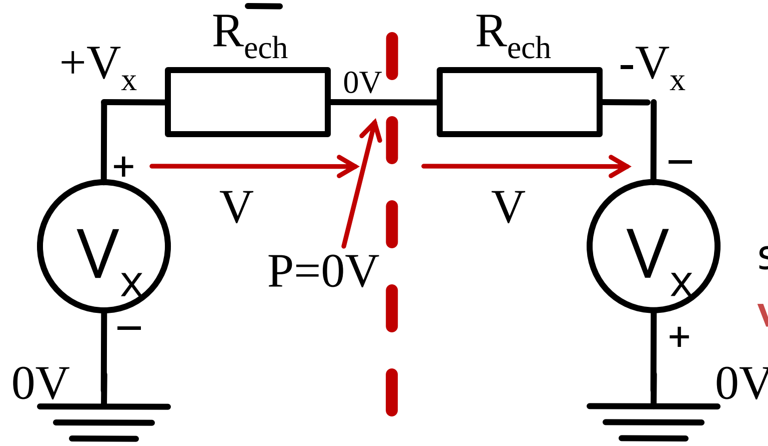




$$I = 0, \forall V_x$$

symmetry plane

open circuit



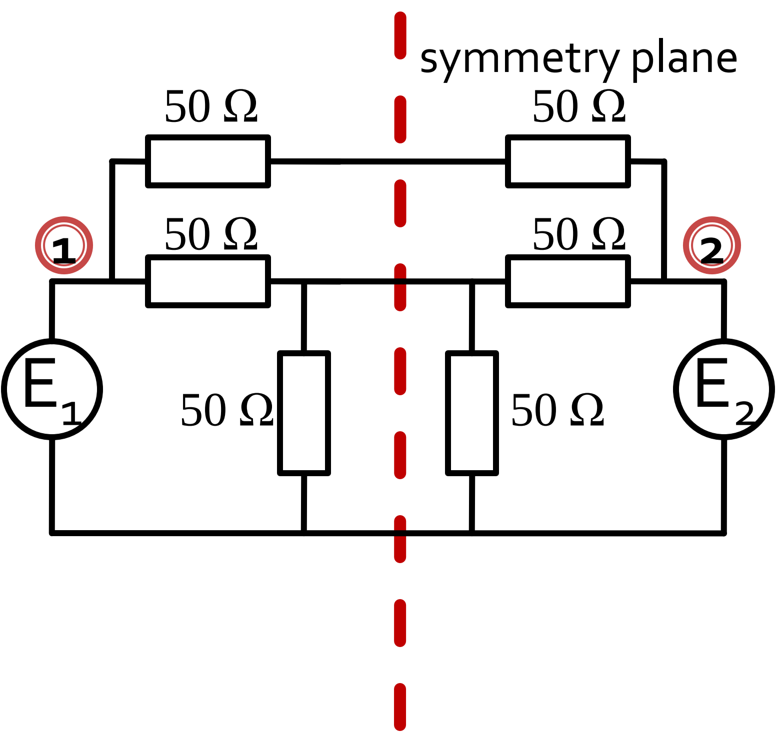
$$P = 0, \forall V_x$$

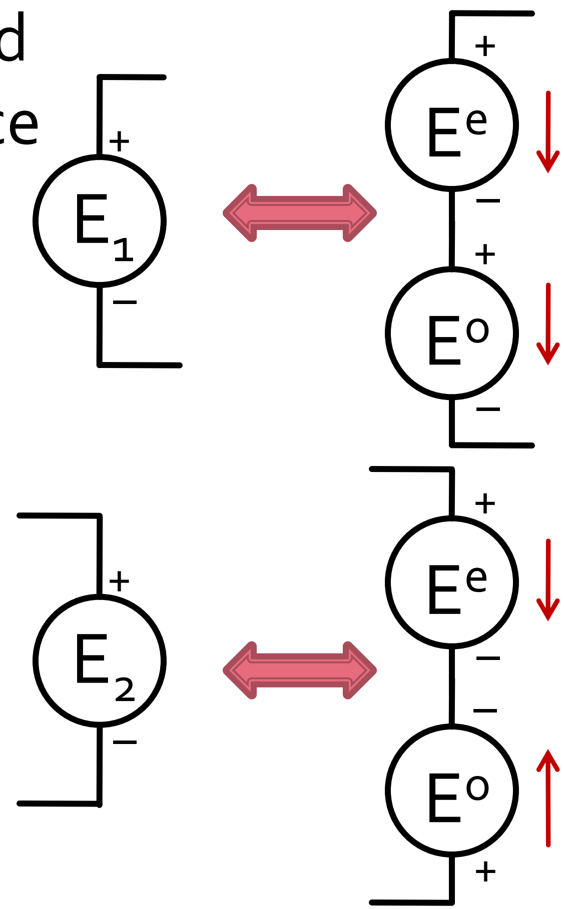
symmetry plane

virtual ground

Even/Odd Mode Analysis

- the combination of any two sources is equivalent for linear circuits with the superposition of:
  - a symmetric source and
  - a anti-symmetric source





$$E_1 = E^e + E^o$$

$$E_2 = E^e - E^o$$

$$E^e = \frac{E_1 + E_2}{2}$$

$$E^o = \frac{E_1 - E_2}{2}$$

Even/Odd Mode Analysis

- In linear circuits the **superposition principle** is always true
  - the response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually

Response (Source1 + Source2) =

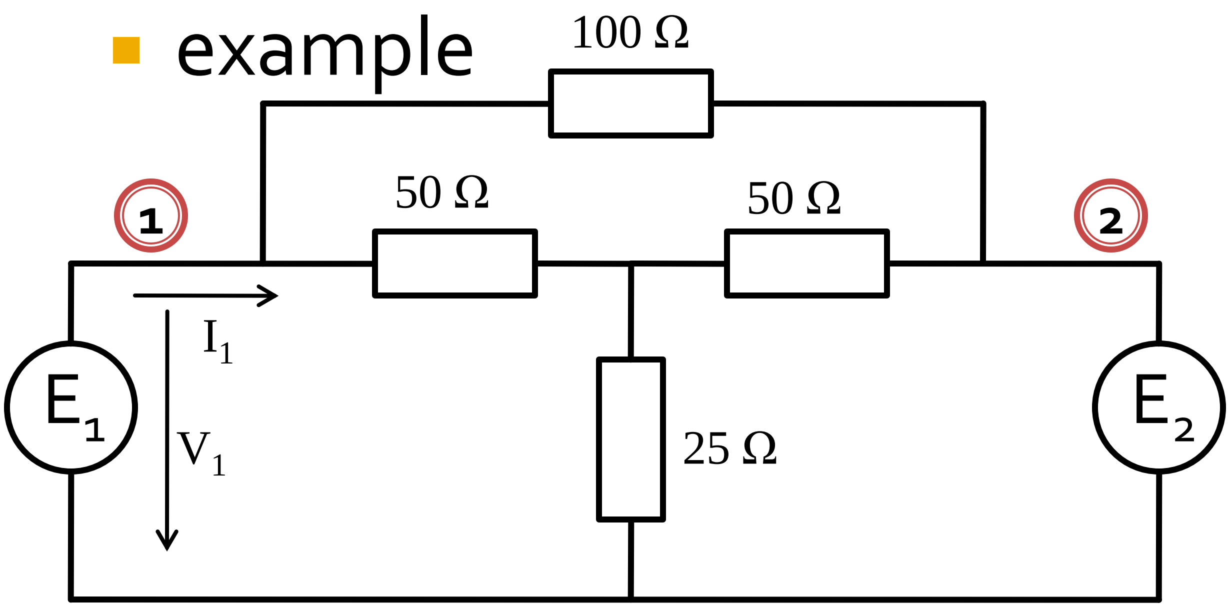
= Response (Source1) + Response (Source2)

Response(ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

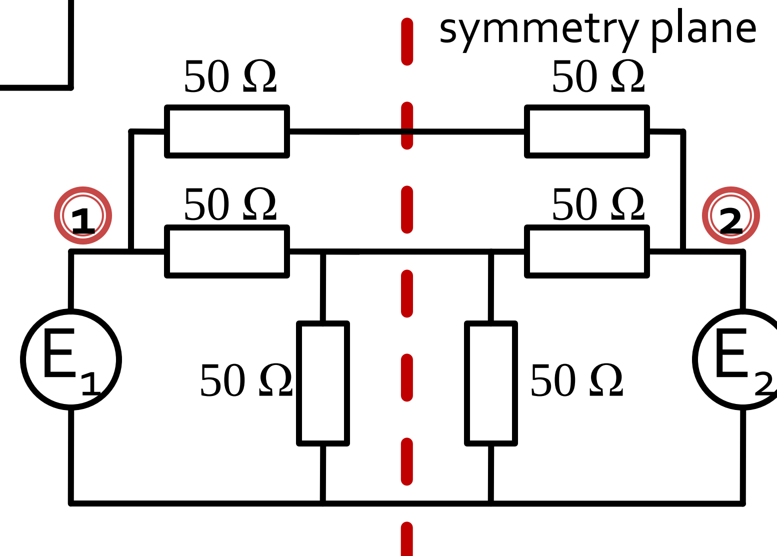
Even/Odd Mode Analysis

- example



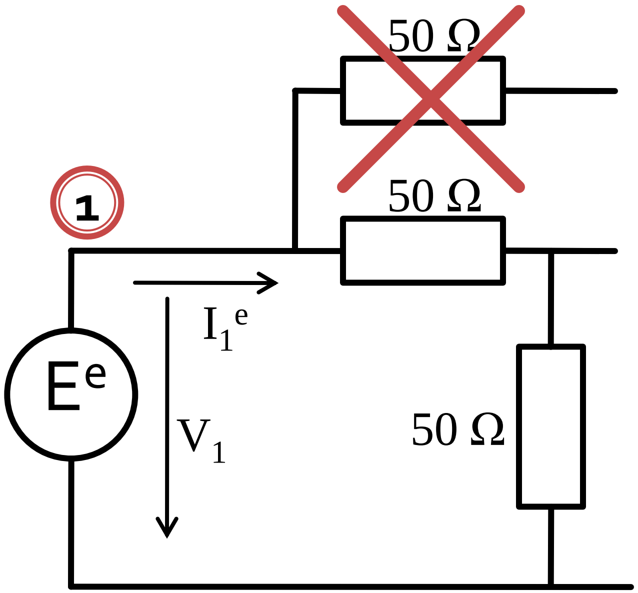
$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$V_2 \equiv E_2 = 0 \Rightarrow \quad E^e = \frac{E_1}{2} \qquad E^o = \frac{E_1}{2}$$



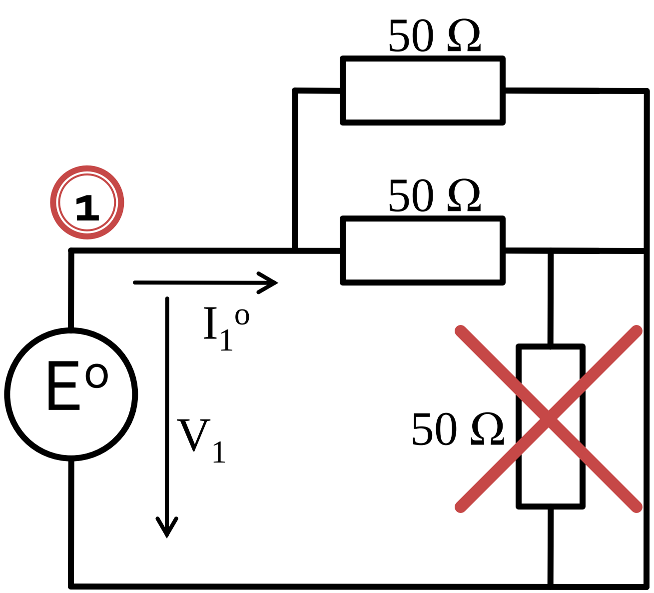
Even/Odd Mode Analysis

- Even/Odd mode analysis



$$R_{ech}^e = 50\Omega + 50\Omega = 100\Omega$$

$$I_1^e = \frac{E^e}{R_{ech}^e} = \frac{E_1/2}{100\Omega} = \frac{E_1}{200\Omega}$$

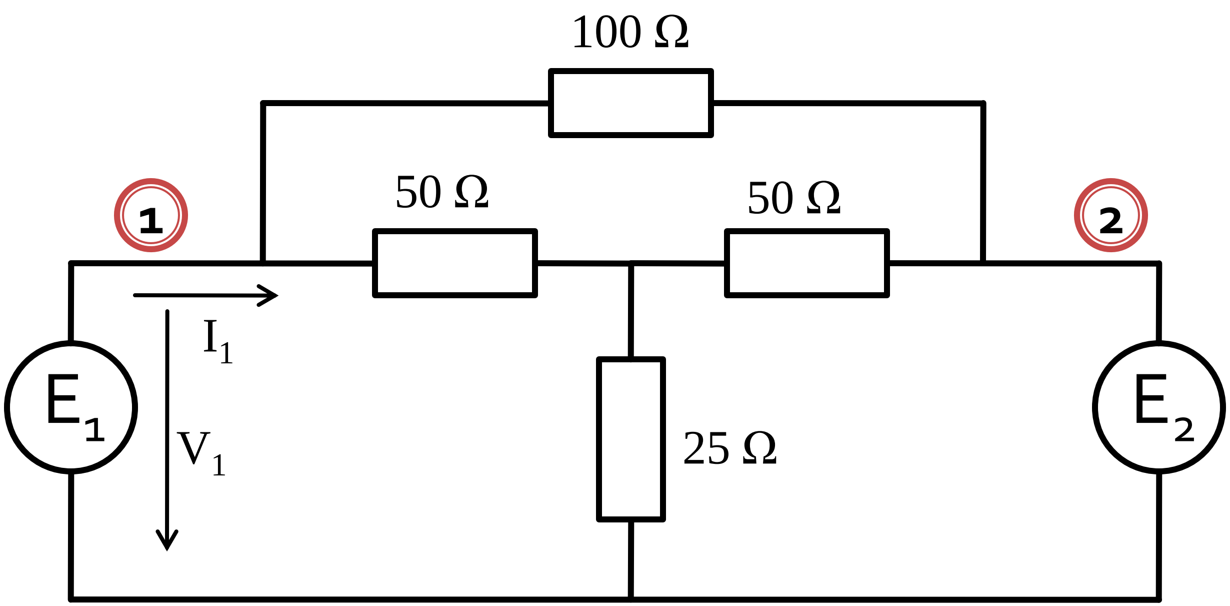


$$R_{ech}^o = 50\Omega || 50\Omega = 25\Omega$$

$$I_1^o = \frac{E^o}{R_{ech}^o} = \frac{E_1/2}{25\Omega} = \frac{E_1}{50\Omega}$$

Even/Odd Mode Analysis

- superposition principle



$$I_1 = I_1^e + I_1^o$$

$$V_1 = V_1^e + V_1^o$$

$$I_1 = I_1^e + I_1^o = \frac{E_1}{200\Omega} + \frac{E_1}{50\Omega} = \frac{E_1}{40\Omega}$$

$$V_1 = V_1^e + V_1^o = E_1$$

$$Y_{11} = \frac{I_1}{V_1} = \frac{1}{40\Omega} = 0.025S$$

Even/Odd Mode Analysis

- In linear circuits we can use the superposition principle
- advantages
  - reduction of the circuit complexity
  - decrease in the number of ports (**main** advantage)

Response(ODD + EVEN) = Response (ODD) + Response (EVEN)

We can benefit from existing symmetries !!

Power dividers and directional couplers

Power dividers and couplers

- Desired functionality:
  - division
  - combining
- of signal power

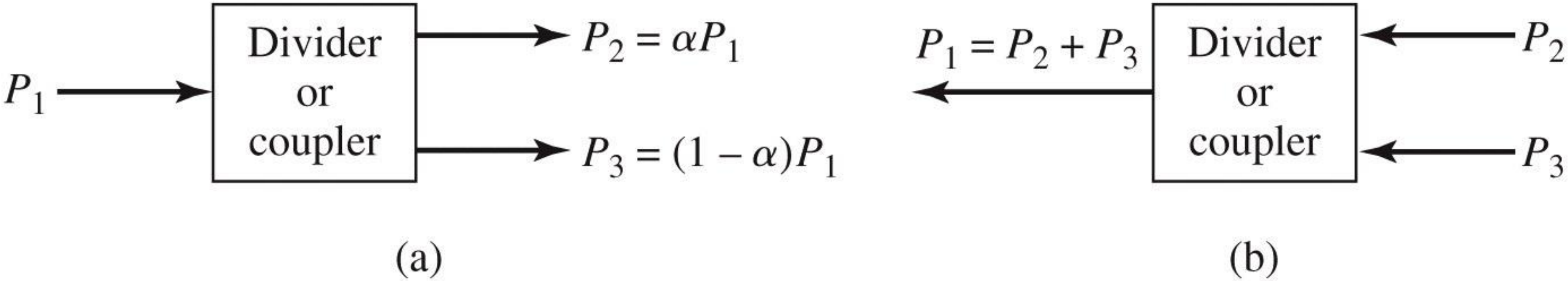


Figure 7.1

Three-Port Networks

- also known as T-Junctions
- characterized by a 3x3 **S** matrix

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

- the device is **reciprocal** if it does **not** contain:
  - anisotropic materials (usually ferrites)
  - active circuits
- to avoid power loss, we would like to have a network that is:
  - **lossless**, and
  - **matched at all ports**
    - to avoid reflection power “loss”

Three-Port Networks

- reciprocal

$$[S]=[S]^t \qquad S_{ij}=S_{ji}, \forall j \neq i$$

$$S_{12}=S_{21}, S_{13}=S_{31}, S_{23}=S_{32}$$

- matched at all ports

$$S_{ii}=0, \forall i \qquad S_{11}=0, S_{22}=0, S_{33}=0$$

- then the S matrix is:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

Three-Port Networks

- reciprocal, matched at all ports, S matrix:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- lossless network
  - all the power injected in one port will be found exiting the network on all ports

$$[S]^{*}\cdot[S]^{t}=[1] \qquad \sum_{k=1}^{N} S_{ki}\cdot S_{kj}^{*}=\delta_{ij},\forall i,j$$

$$\sum_{k=1}^{N} S_{ki}\cdot S_{ki}^{*}=1 \qquad \sum_{k=1}^{N} S_{ki}\cdot S_{kj}^{*}=0,\forall i\neq j$$

Three-Port Networks

- lossless network

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

$$\sum_{k=1}^{N} S_{ki}\cdot S_{ki}^{*}=1$$

$$\sum_{k=1}^{N} S_{ki}\cdot S_{kj}^{*}=0, \forall i\neq j$$

- 6 equations / 3 unknowns

$$|S_{12}|^2+|S_{13}|^2=1 \qquad S_{13}^{*}S_{23}=0$$

$$|S_{12}|^2+|S_{23}|^2=1 \qquad S_{12}^{*}S_{13}=0$$

$$|S_{13}|^2+|S_{23}|^2=1 \qquad S_{23}^{*}S_{12}=0$$

- no solution is possible

Three-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

- 6 equations / 3 unknowns
  - no solution is possible
- A three-port network **cannot** be simultaneously:
  - reciprocal
  - lossless
  - matched at all ports
- If any one of these three conditions is relaxed, then a physically realizable device is possible

Nonreciprocal Three-Port Networks

- usually containing anisotropic materials, ferrites
- nonreciprocal, but matched at all ports and lossless $S_{ij} \neq S_{ji}$
- S matrix

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

- 6 equations / 3 unknowns

$$|S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{31}^* S_{32} = 0$$

$$|S_{21}|^2 + |S_{23}|^2 = 1 \qquad S_{21}^* S_{23} = 0$$

$$|S_{31}|^2 + |S_{32}|^2 = 1 \qquad S_{12}^* S_{13} = 0$$

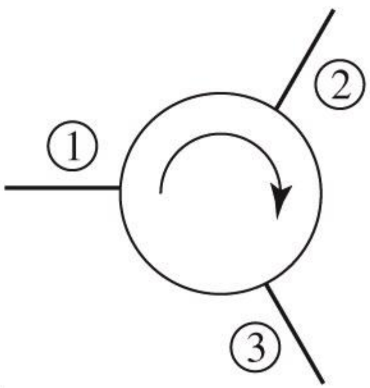
Nonreciprocal Three-Port Networks

- two possible solutions
- circulators
  - clockwise circulation

$$S_{12} = S_{23} = S_{31} = 0$$

$$|S_{21}| = |S_{32}| = |S_{13}| = 1$$

$$[S] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



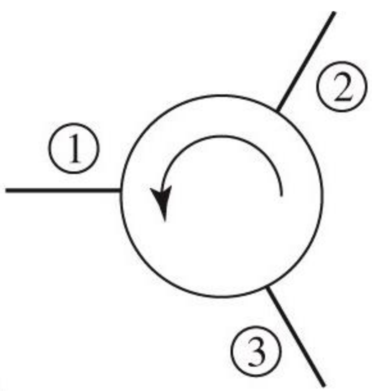
(a)

  - counterclockwise circulation

$$S_{21} = S_{32} = S_{13} = 0$$

$$|S_{12}| = |S_{23}| = |S_{31}| = 1$$

$$[S] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$



(b)

Mismatched Three-Port Networks

- A lossless and reciprocal three-port network can be matched only on two ports, eg. 1 and 2:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

$$S_{13}^{*}S_{23}=0$$

$$S_{12}^{*}S_{13}+S_{23}^{*}S_{33}=0$$

$$S_{23}^{*}S_{12}+S_{33}^{*}S_{13}=0$$

$$|S_{12}|^2+|S_{13}|^2=1$$

$$|S_{12}|^2+|S_{23}|^2=1$$

$$|S_{13}|^2+|S_{23}|^2+|S_{33}|^2=1$$

$$S_{13}=S_{23}=0$$

$$|S_{13}|=|S_{23}|$$

$$|S_{12}|=|S_{33}|=1$$

Mismatched Three-Port Networks

- A lossless and reciprocal three-port network

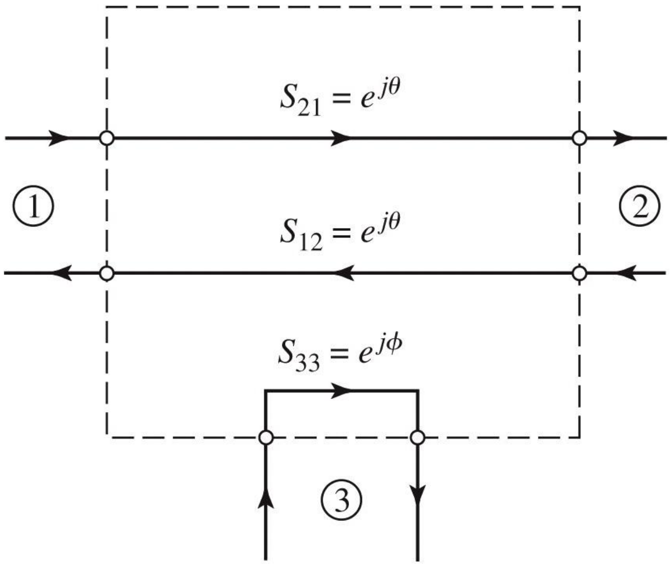
$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

$$S_{13}=S_{23}=0 \qquad |S_{12}|=|S_{33}|=1$$

$$S_{12}=e^{j\theta}$$

$$S_{33}=e^{j\varphi}$$

$$[S]=\begin{bmatrix} 0 & e^{j\theta} & 0 \\ e^{j\theta} & 0 & 0 \\ 0 & 0 & e^{j\phi} \end{bmatrix}$$



- A lossless and reciprocal three-port network **degenerates** into two separate components:
  - a matched two-port **line**
  - a totally **mismatched one-port**:

Four-Port Networks

- characterized by a 4x4 **S** matrix

$$[S]=\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

- the device is **reciprocal** if it does **not** contain:
  - anisotropic materials (usually ferrites)
  - active circuits
- to avoid power loss, we would like to have a network that is:
  - **lossless**, and
  - **matched at all ports**
    - to avoid reflection power "loss"

Four-Port Networks

- reciprocal

$$[S]=[S]^t \qquad S_{ij}=S_{ji},\forall j\neq i$$

$$S_{12}=S_{21}, S_{13}=S_{31}, S_{23}=S_{32}$$

- matched at all ports

$$S_{ii}=0,\forall i \qquad S_{11}=0, S_{22}=0, S_{33}=0, S_{44}=0$$

- then the S matrix is:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

Four-Port Networks

- reciprocal, matched at all ports, S matrix:

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

- lossless network
  - all the power injected in one port will be found exiting the network on all ports

$$[S]^* \cdot [S]^t = [1] \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = \delta_{ij}, \forall i, j$$

$$\sum_{k=1}^{N} S_{ki} \cdot S_{ki}^* = 1 \qquad \sum_{k=1}^{N} S_{ki} \cdot S_{kj}^* = 0, \forall i \neq j$$

Four-Port Networks

$$S_{13}^* \cdot S_{23} + S_{14}^* \cdot S_{24} = 0 \quad / \cdot S_{24}^*$$
$$S_{14}^* \cdot S_{13} + S_{24}^* \cdot S_{23} = 0 \quad / \cdot S_{13}^*$$
$$\overline{S_{14}^* \cdot \left(|S_{13}|^2 - |S_{24}|^2 \right) = 0}$$

$$S_{12}^* \cdot S_{23} + S_{14}^* \cdot S_{34} = 0 \quad / \cdot S_{12}$$
$$S_{14}^* \cdot S_{12} + S_{34}^* \cdot S_{23} = 0 \quad / \cdot S_{34}^*$$
$$\overline{S_{23} \cdot \left(|S_{12}|^2 - |S_{34}|^2 \right) = 0}$$

- one solution: $S_{14} = S_{23} = 0$
- resulting coupler is **directional**

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$|S_{12}|^2 + |S_{13}|^2 = 1$$
$$|S_{12}|^2 + |S_{24}|^2 = 1$$
$$|S_{13}|^2 + |S_{34}|^2 = 1$$
$$|S_{24}|^2 + |S_{34}|^2 = 1$$

$$|S_{13}| = |S_{24}|$$

$$|S_{12}| = |S_{34}|$$

Four-Port Networks

$$[S]=\begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$|S_{12}|=|S_{34}|=\alpha \qquad |S_{13}|=|S_{24}|=\beta$$

β – voltage coupling coefficient

- We can choose the phase reference

$$S_{12}=S_{34}=\alpha \qquad S_{13}=\beta\cdot e^{j\theta} \qquad S_{24}=\beta\cdot e^{j\phi}$$

$$S_{12}^{*}\cdot S_{13}+S_{24}^{*}\cdot S_{34}=0 \quad \rightarrow \quad \theta+\phi=\pi\pm 2\cdot n\cdot\pi$$

$$|S_{12}|^{2}+|S_{24}|^{2}=1 \quad \rightarrow \quad \alpha^{2}+\beta^{2}=1$$

- The other possible solution for previous equations offer either essentially the same result (with a different phase reference) or the degenerate case (2 separate two port networks side by side)

$$S_{14}^{*}\cdot\left(|S_{13}|^{2}-|S_{24}|^{2}\right)=0 \qquad S_{23}\cdot\left(|S_{12}|^{2}-|S_{34}|^{2}\right)=0$$

Four-Port Networks

- A four-port network simultaneously:
  - matched at all ports
  - reciprocal
  - lossless
- is **always directional**
  - the signal power injected into one port is transmitted **only towards two** of the other three ports

$$[S]=\begin{bmatrix} 0 & \alpha & \beta\cdot e^{j\theta} & 0 \\ \alpha & 0 & 0 & \beta\cdot e^{j\phi} \\ \beta\cdot e^{j\theta} & 0 & 0 & \alpha \\ 0 & \beta\cdot e^{j\phi} & \alpha & 0 \end{bmatrix}$$

Four-Port Networks

- two particular choices commonly occur in practice
  - A Symmetric Coupler $\theta = \phi = \pi/2$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

  - An Antisymmetric Coupler $\theta = 0, \phi = \pi$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

Directional Coupler

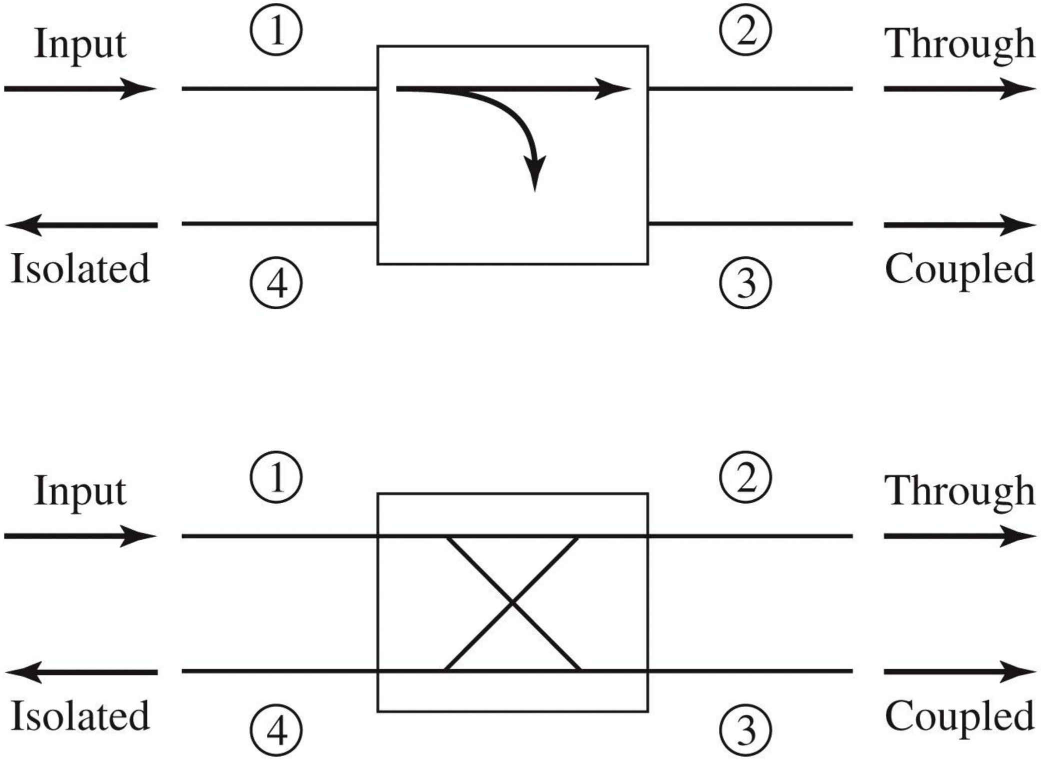


Figure 7.4

$$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$

$$|S_{13}|^2 = \beta^2$$

Coupling

$$C = 10\log\frac{P_1}{P_3} = -20\cdot\log(\beta)\,[\text{dB}]$$

Directivity

$$D = 10\log\frac{P_3}{P_4} = 20\cdot\log\left(\frac{\beta}{|S_{14}|}\right)[\text{dB}]$$

Isolation

$$I = 10\log\frac{P_1}{P_4} = -20\cdot\log|S_{14}|\,[\text{dB}]$$

$$I = D + C\,,\ \ dB$$

Directional Couplers

Laboratory no. 2

Directional Coupler

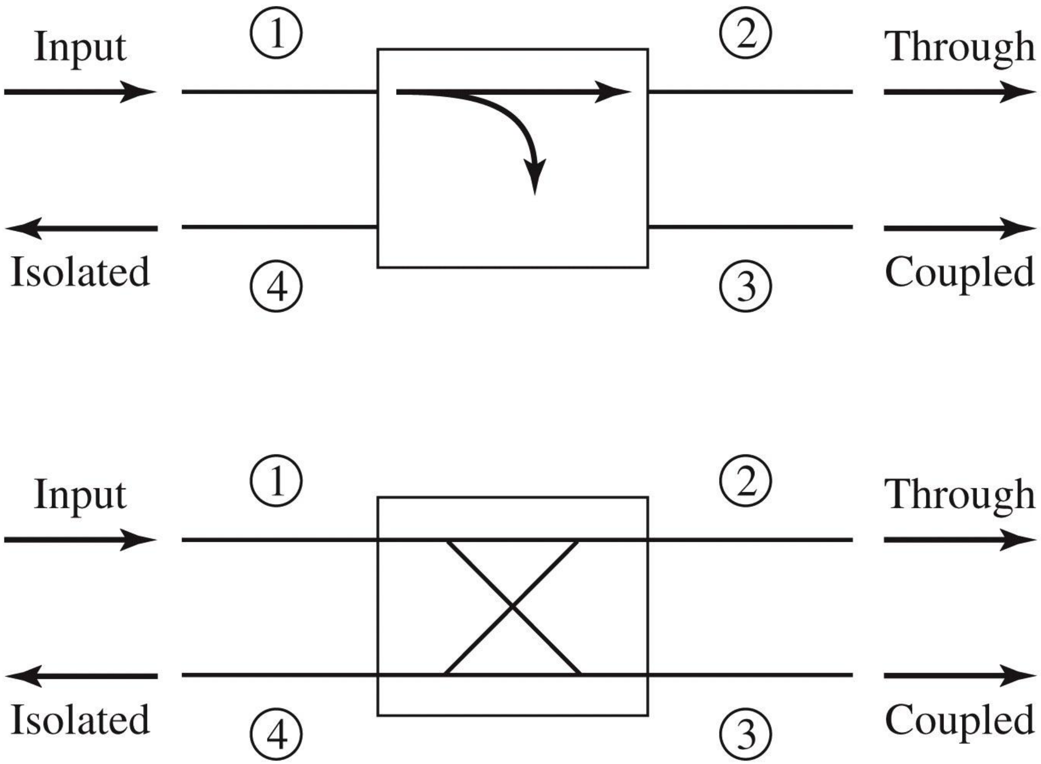


Figure 7.4

$$|S_{12}|^2 = \alpha^2 = 1 - \beta^2$$

$$|S_{13}|^2 = \beta^2$$

Cuplaj

$$C = 10 \log \frac{P_1}{P_3} = -20 \cdot \log(\beta) \, [\text{dB}]$$

Directivitate

$$D = 10 \log \frac{P_3}{P_4} = 20 \cdot \log\left(\frac{\beta}{|S_{14}|}\right) [\text{dB}]$$

Izolare

$$I = 10 \log \frac{P_1}{P_4} = -20 \cdot \log|S_{14}| \, [\text{dB}]$$

$$I = D + C \,, \;\; dB$$

Quadrature coupler

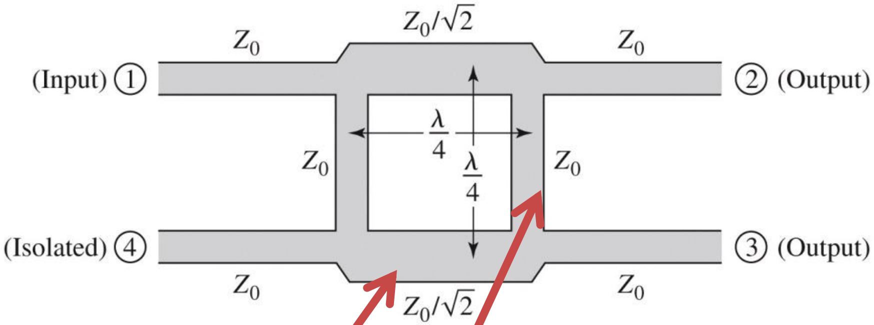


Figure 7.21

$$y_2^2 = 1 + y_1^2$$

$$|\beta| = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$C[\mathrm{dB}] = -20 \cdot \log_{10} \frac{\sqrt{y_2^2 - 1}}{y_2}$$

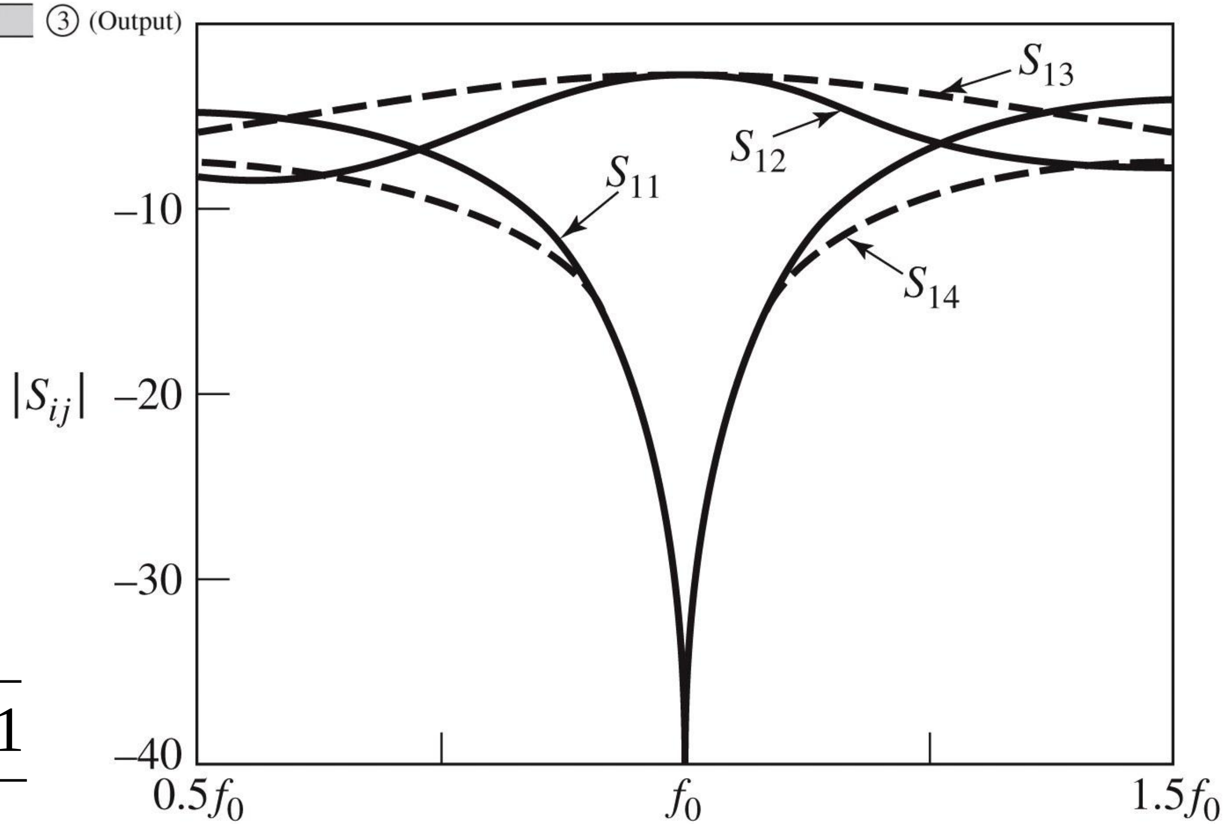
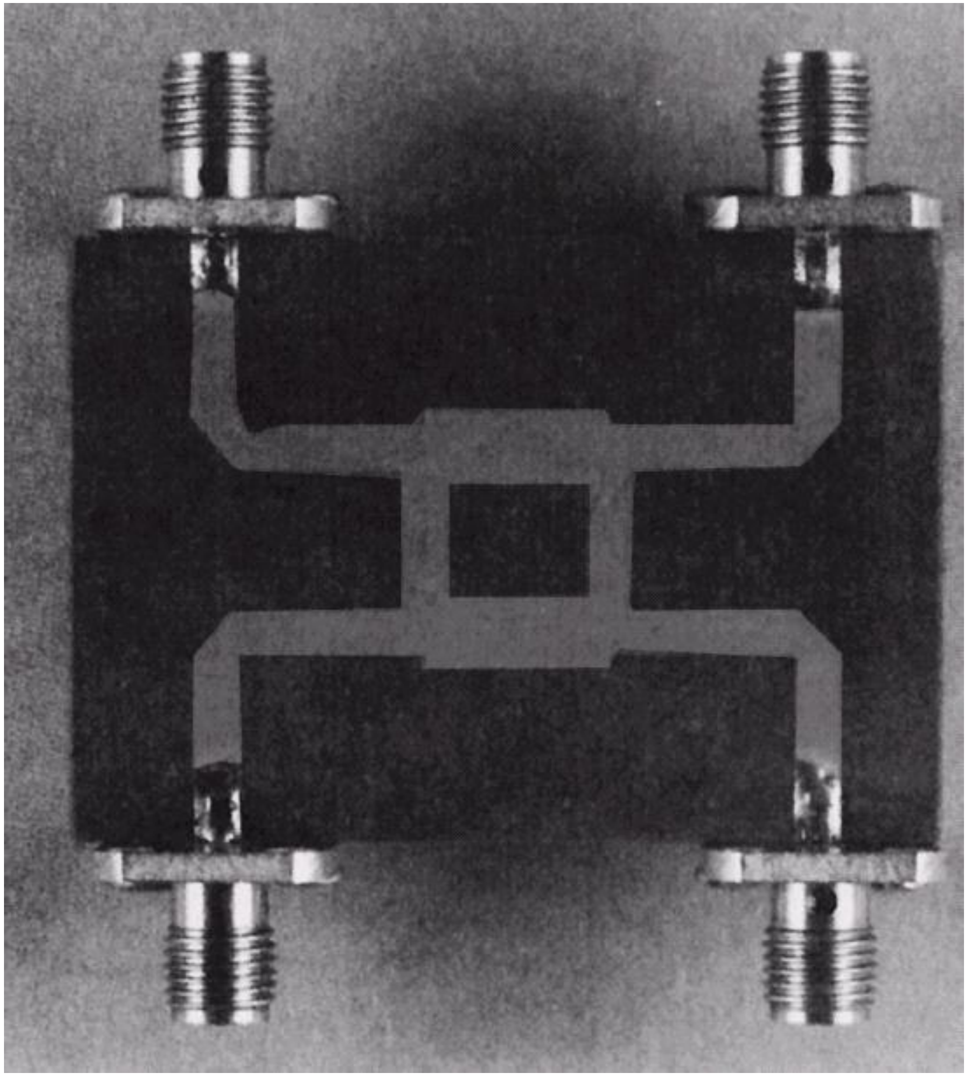
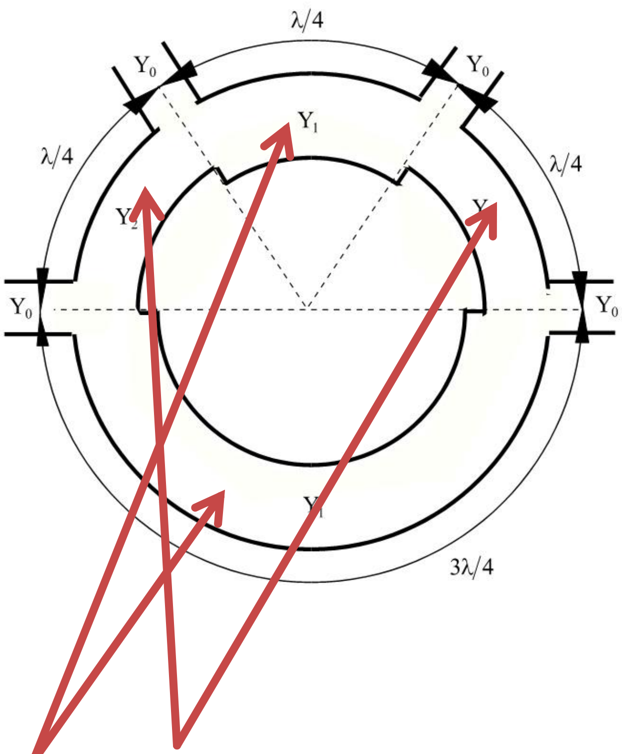


Figure 7.25

Quadrature coupler



Ring coupler



$$y_1^2 + y_2^2 = 1$$

$$|\beta| = y_1$$

$$C\,[\mathrm{dB}] = -20 \cdot \log_{10}(y_1)$$

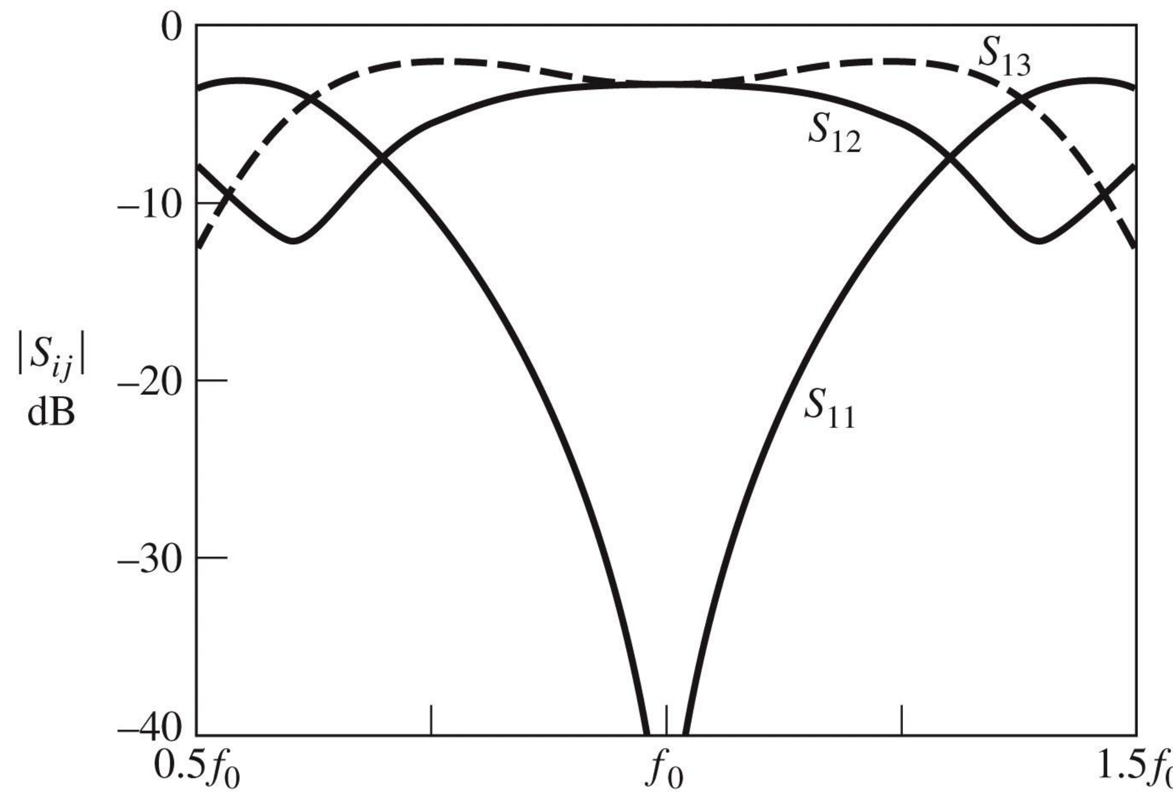


Figure 7.46

Ring coupler

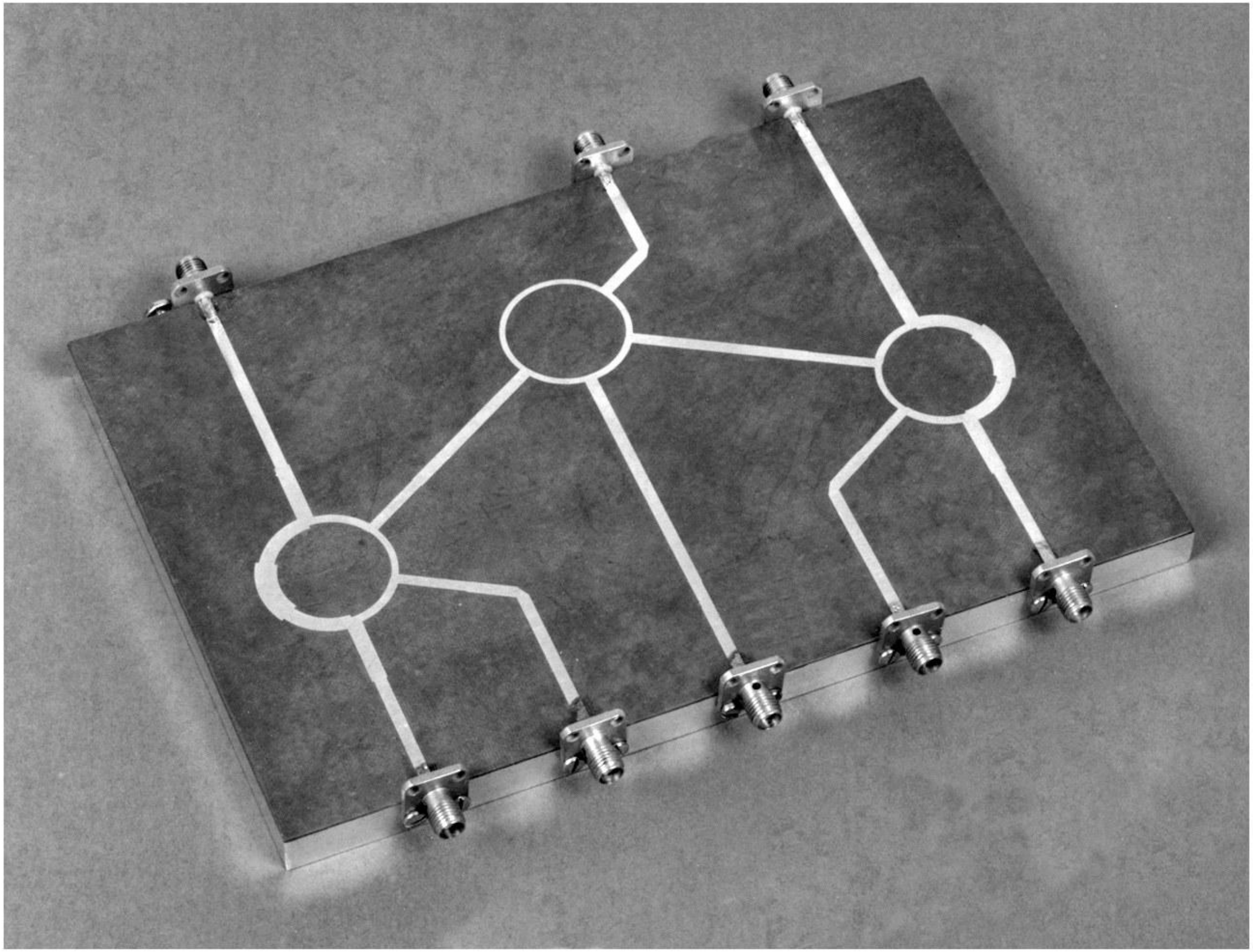
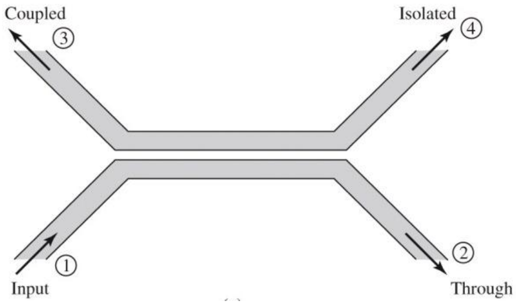


Figure 7.43
Courtesy of M. D. Abouzahra, MIT Lincoln Laboratory, Lexington, Mass.

Coupled line coupler



$$Z_{ce}Z_{co} = Z_0^2$$

$$|\beta| = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$C\,[\mathrm{dB}] = -20 \cdot \log_{10}\left(\frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}\right)$$

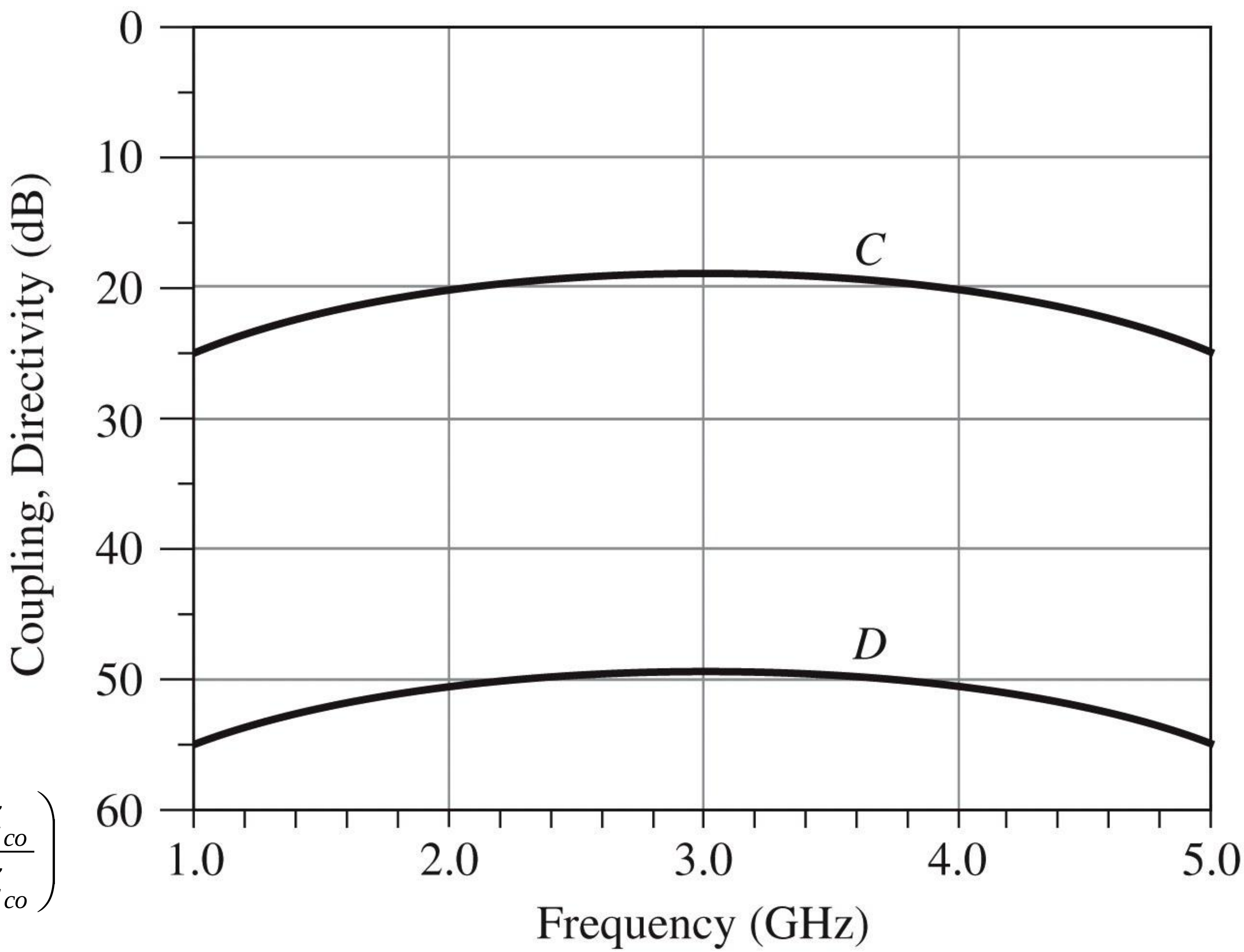
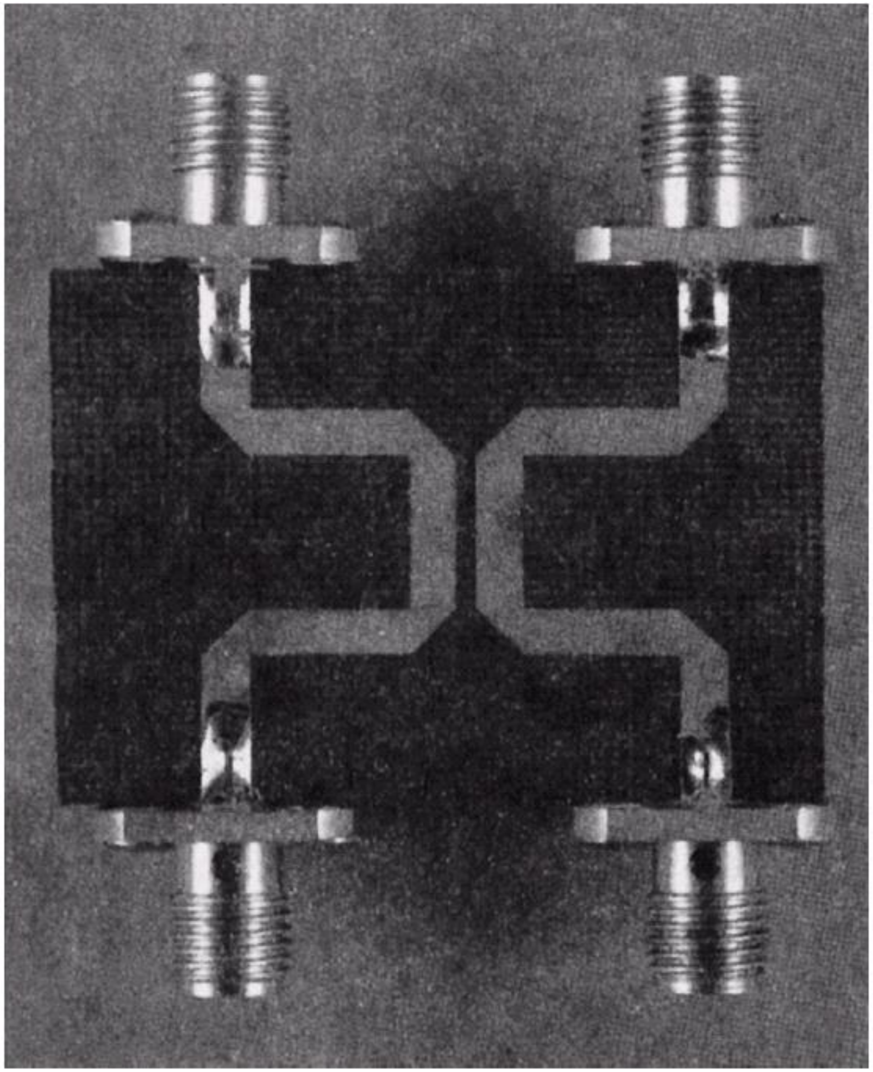


Figure 7.34

Coupled line coupler



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